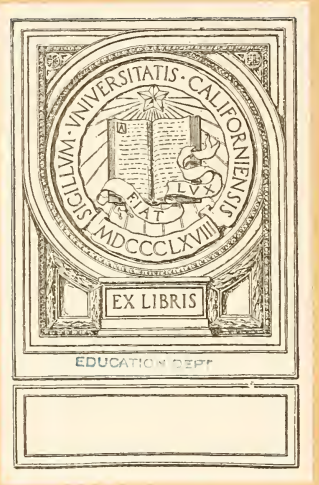
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A study of eliminations, inclusions, and social and business requirements of arithmetic

By

Katherine Spiers

B.L. 1914

THESIS

Submitted in partial satisfaction of the requirements for the degree of

MASTER OF ARTS

in

Education

in the

GRADUATE DIVISION

of the

UNIVERSITY OF CALIFORNIA

Dec. 1921

Approved C. D. MEAD
Instructor in Charge

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INTRODUCTION.

The thesis I maintain is this:

The kind and amount of arithmetic taught in the elementary school and the method of its teaching rest upon the demands of the business and the social world.

In establishing this thesis I shall approach the problem in two ways: first, by examining the accessible writings of recognized educators and by collecting, collating and organizing the records of what has been done by investigators up to this time in meeting the needs of society, in setting up aims, in determining essentials, in eliminating useless subject-matter, in proposing methods and means of procedure, and in compiling practical courses of study; second, by first hand investigation and study of the requirements of society, I shall endeavor to search out the minimum essentials of arithmetic.

Using the data, records and judgments collected under the first section of this study as a basis for the comparison and testing of the later findings, I hope to determine in what ways and to what extent the subject-matter may be further simplified, in how far the bulk of material may be reduced, and how great a decrease in the time devoted to the teaching of arithmetic may be made.

By inquiring into current business practices, I shall

make an effort to offer a harmonization of school methods and world methods of procedure, and shall suggest certain inclusions demanded by social needs.

Finally, I shall include a course of study in arithmetic founded upon the facts and truths established.

CHAPTER I

THE NEED FOR SCIENTIFIC INVESTIGATIONS IN EDUCATION.

"There is no form of knowledge so complete and final that it cannot be improved, no single human art so perfect that it cannot be made better, no form of human endeavor that does not call for further effort. For this philosophy, life is a perfecting, not an arriving at perfections, and the joy is in the process, not in reaching and remaining at the goal."[1]

The best teaching of truth is that which seeks to define not truth itself as something already complete, but the nature of the effort required to search it out.

The birth and growth of the idea of investigations, educational and social surveys, and general stock-taking for every sort of institution has been within the lifetime of the present generation. The idea did not spring into existence full-formed, but is developing gradually, and is marked along the way by revolutionary principles. This movement in the elementary schools is of special significance, for elementary school education has fully entered the experimental period. Rapid changes are being made in school subjects through the influence of modern scientific research. The initial need is to know whither our present school system is taking us, and if by critical examination it be determined that our direction is wrong, we must send out e uca-

1. Moore, Ernest Carroll: What is Education? 1914, page 141.

tional scouts to spy out direct roads to a pioneer land where we may find the truth.

What we have in education is an inheritance from our forefathers. It is the product of indiscriminate borrowing, modified as need forced modifications upon us. Frank experimentation is imperative. Our generation must shake off tradition and organize a sound scheme for educational advancement. Changes must not be made according to the whims of individuals -- the "cut-and-paste" method by which the average school superintendent compiles his course of study is tragically common -- nor must these changes be made by local committees transient in character. Only after theories have been proved by investigation according to scientific method, and have been successfully tested and tried in the school of practice, may they be pronounced true and good.

Many problems pertaining to the construction of courses of study already have been solved, and an encouraging body of scientific data has been secured. Many foolish things, no doubt, have been done under the shield of experimentation, but not adhered to by the conscientious worker in the field. He may make mistakes, but he discovers and corrects them when he tests his theories by practice. The markets are stocked with books, and eager teachers, with more enthusiasm than wisdom, make sporadic trial of this and that new idea. Opinion serves for information, and the disastrous results often bring disrepute upon a system

of attack which is striving zealously to advance education. Comparisons must be made between the conclusions reached in original research, and the results obtained by previous investigators along a like line before judgments can be made. And then, that no reforms may be offered which have not stood the test of our own trying-ground, every fact advanced must stand the test of application; for the only effective way to determine whether the conclusions arrived at by experimental study are feasible is to see that they work out well in practice under the conditions of the classroom. There must be no gap between educational experimentation and school practice. Fortunately, in the school world, there is a general awakening to a sense of values based upon scientific analysis. By discovery, surveys, facing and solving problems which individuals and communities have to meet, we are led away from the dogmatical, the formal, and the untrue.

Our investigations and experimentations must be based upon sound educational aims. To make the discovery of anything worth while, we must know what human purpose is to be served by its discovery. What is our problem? What is our purpose? What are we trying to do? What is our Aim? We must have no groping. As General Foch said, "There is but one manner of considering every question, that is the objective manner." What is our Objective?

Aims in education established by masters seem to vary: Knowledge getting, self-realization, social efficiency, culture,

achievement, adjustment, learning to use the tools which the race has found indispensable, duty, growth, utility, complete living, discipline, -- how each aim calls to mind the name of its advocate, name and aim closely locked. -- Christ's, "I am come that they might have life, and that they might have it more abundantly," is an educational aim. These ideals established by profound thinkers seem so numerous one is overwhelmed when confronted by them. Have these scholars who have spent their years examining education arrived at diametrically different conclusions? Are self-realization, efficiency, culture, achievement, complete living, utility, growth, aims separate and apart? May they not be aspects of one great requirement? May not the motif running through all be, "We serve life"?

Broadly speaking, we may shift advocated educational aims into three general groups: knowledge-getting, discipline, use.

Knowledge getting, when knowledge for the sake of knowledge is the aim, is an old, an established aim. With this the end of education, the seeker is prescribed facts regardless of the usefulness of these facts. The schools are knowledge dispensaries where facts are parcelled out by those who make knowledge bestowing a business. The admonition, "Hold fast all I give you," accompanies the largess, and a strict accounting of the accumulated information is required. "Remember and you are educated," is the motto. Arithmetic facts, geography facts, spelling facts,

history facts, literature, art and language facts, thousands and tens of thousands of them compounded are forced upon the trusting ones who come asking for that which will cure their ignorance. Up to the past twenty-five years teachers universally accepted the decree of tradition, and made little effort to select usable from useless facts. They, with the aid of a few standardized texts, prodigally passed out facts. The key-word was, "Know for the sake of knowing." "How much do you know?" not, "What can you do with what you know?" is the shibboleth of those who lay the supreme emphasis upon the acquisition of knowledge as the end of education.

The modern educational world is striving to overcome this doctrine. The ideals of such leaders as McMurry,[1] Cubberley,[2] Dewey,[3] Hall,[4] O'Shea,[5] Moore,[6] Thorndike[7] and others, are becoming directing forces. Dewey, for example, utterly opposes information as the end of education. He deprecates the accumulation of facts for recitation, and the hoarding of knowledge. "Knowledge

Characteristic Works:

1. McMurry, Frank: Elementary School Standards. 1913.
2. Cubberley, Ellwood P.: Changing Conceptions of Education. 1909.
3. Dewey, John: Democracy and Education. 1916
4. Hall, G. Stanley: Adolescence. 1904. Educational Problems. 1911.
5. O'Shea, Vincent: Education as Adjustment. 1903.
   Dynamic Factors in Education. 1906
6. Moore, Ernest C.: What is Education? 1914.
7. Thorndike, Edward Lee: Educational Psychology. 1914.

in the sense of information, means the working capital, the indispensable resources, of further inquiry; of finding out, or learning more things. Frequently it is treated as an end in itself, and then the goal becomes to heap it up and display it when called for. This static, cold-storage ideal of knowledge is inimical to educative development."[1] It is particularly mischievous, he holds, because the cramming of the mind with non-pertinent facts not only lets occasions for thinking go unused, but swamps thinking.

Moore,[2] believes that knowledge simply for knowledge sake is impractical; it is neither warranted by facts, nor by psychological tests. The purpose of the investigator is to acquire knowledge, but when the facts are in his possession they must meet the question, "What are they worth?" Since we are primarily concerned with living rather than knowing, we select those things which constitute vital knowledge. Knowledge as the power to act excludes all useless information.

Teachers and administrators are profiting through an examination of the work done in the Francis Parker, and the Horace Mann schools. Schools are being looked upon as places where new ideas may be evolved; and new discoveries made and tested.

The ideal, knowledge for knowledge sake, which has

1. Dewey, John: Democracy and Education, page 185.
2. Moore, Ernest C.: What is Education? Chapter II.

played so prominent a part in determining the curricula and the methods in our schools, which has evaluated studies according to their ability to impart information, is giving place to that knowledge which arises from use and is for use.

The doctrine of general education, or formal discipline, is a second heritage from the past. The belief that training received in one line of mental activity spreads to other lines of mental activity is behind much of what we do in elementary schools, in high schools, and in colleges. This great assumption is the one upon which education has rested for many centuries. Studies, according to the advocates of this theory, have magic powers. They discipline, develop, and perfect minds. They are to be pursued not because they have specific values, but because they improve the mental faculties. Observation, emotions, reason, will, memory can be trained in wholesale fashion. It is the schoolmaster's place to make of his classroom a mental gymnasium where his pupils' minds, by daily exercises upon the apparatus furnished, may be made more agile, keener, stronger. The more difficult the apparatus is, the finer, abler, and more nearly perfect the resulting mind will be. Greek, Latin and higher mathematics have ranked high in this educational gymnasium. A Marathon through some unapplied science, a strenuous wrestling bout with rhetoric, a pugilistic encounter with algebra, daily trying-out tournaments with some hundred or so of the 400,000 words of our English speech are supposed to increase mind power. The

gentler calisthenics of poetry, art, and music will lend grace. Here, too, imperfect minds may be toned-up, burnished, repaired. The different types of minds are little considered and the pupils, in groups, pass with lock-step regularity from exercise to exercise throughout the day.

If we turn to experimental studies made upon this subject, and judge by the writings of scholars[1], in the past twenty-five years, we cannot fail to recognize the error of this doctrine. The data of those° whose bias is frankly favorable to the theory seem the most conclusive against it. But old custom has imbedded it. Perhaps as many as three-fourths of the teachers of to-day, and undoubtedly as many as nine-tenths of the educated parents adhere to it as firmly as though faculty psychology had never been questioned. But Schools of Education and research workers are making headway. The newer West seems freer of this pedantry than the older East. This confusion which began in that far-away day when the Sophists taught that if one wanted to be a physician he must study rhetoric, and if he wanted to be a general he must study speech-making must be completely cleared away.

1. Dewey, John: Democracy and Education.
   Moore, Ernest Carroll: What is Education? 1914.

° Rugg, H.O.: The Experimental Determination of Mental Discipline in School Studies. 1916
Moore, Charles N.: The Inadequacy of Arguments against Disciplinary Values. School and Society. Dec.29,1917.
Coover, John Edgar: Formal Discipline from the Standpoint of Experimental Psychology. 1916.

Whipple says, "The problem of mental discipline, of determining under what conditions, by what methods, and to what extent training in a given line of mental activity extends to other lines of mental activity is acknowledged to be the central problem of educational psychology."[1] Moore says, "Formal discipline is the central problem of educational philosophy, and the attitude which we who teach take upon this problem determines, as nothing else does, what we put into courses of study, and how we teach that which we attempt to teach."[2]

This doctrine, undoubtedly a pernicious and painful error, will go the way of wornout superstitions only when the ancient idol has been completely demolished by painstaking research.

Use, the third educational aim, we hold is education's goal. Our real reason for including studies in our curriculum is that they are indispensable helps to us in life. Let our words to our classes be, "You are here to learn to do certain things which the race has found that it cannot live without doing. Every lesson has a specific aim which you are first to see, and then if possible to accomplish. The question for you at all stages is, 'Can you do these socially necessary things?'"[3] Here,

1. Whipple, G. M.: Preface to Ruggs' "Experimental Determination of Mental Discipline in School Studies."
2. Moore, E.C.: Address to Superintendents, State Normal School. Feb. 9, 1918.
3. Moore, E.C.: Address to Superintendents, Los Angeles State Normal. Feb. 9, 1918.

our schoolroom is a work shop where children learn to use the tools which the race has found indispensable. It is of immeasurable importance that the things which a child spends his time upon shall be of immediate use, and shall serve in the future in such a way that he shall go on using them and increasing his mastery of them. We must examine the needs of the adult world and aid boys and girls to acquire the beginnings of the knowledge and the skills demanded for the future.

The writer, by means of personal interviews, endeavored to discover from the people themselves what was their purpose in sending their boys and girls to school. "What do you expect the schools to do for your child?" was the essence of the questions asked of one hundred parents in many different walks in life. College professors, ministers, bankers, physicians, ranchers, plumbers, firemen, grocers, shoemakers, men and women in offices, shops and stores, truck drivers, the scissors grinder, the garbage collector, and the Chinese green grocer were among those questioned. The reasons were varied; indeed, almost as numerous were they as the people who offered them:

"To train his mind."

"To learn the things he will need to know to get on in life."

"To train his reasoning powers."

"To get knowledge."

"So he will be cultured."

"To teach him to live with his fellows, and to pick up a bit of knowledge as he goes along."

"To teach him to think logically."

"To learn to speak and write nicely."

"I don't know. I've never thought about it. Everybody sends children to school. They have to."(Custom.)

"So he may help me with the store, and read and write letters."

"So he will know more and make a better living than I have made."

It was interesting to note that the rich and educated parents asked for knowledge, discipline, culture. As one went down the financial and social scale, the demand for a useful education increased. Products of colleges and schools cling to the traditional reasons for educating; the more uneducated classes claim practical reasons.

Tabulated the results stand:

1. Skills, and usable knowledge	39
2. General education -- discipline	27
3. Knowledge getting.	16
4. Culture (four of these college professors.)	8
5. Social adjustment	7
6. Custom.	3
Total	100

These data are offered as an interesting showing of the public's reaction to a mutable question, not as proof of anything.

If we accept use as our standard, spending a proportional fraction of each day in the company of reading, writing, arithmetic, geography, history, et cetera, will not do. We must select the studies and the parts of the studies which we emphasize; there must be a purposive accumulation of facts. Our sole aim in learning anything must be to learn to use it. Plato has phrased it "teaching the young the knowledge which they will afterwards require for their arts."

In this educational turmoil there is one confusion we must keep clear of. When we say all training shall be specific, we do not mean that education shall be narrow or circumscribed, nor that subjects shall be walled off into compartments. Instruments in the use of which we have acquired skill must be constantly available: our whetted axe is a general tool. If we are to substitute the direct attack for the superstitious routine, our first step must be to gather the tools which will contribute to that result.

CHAPTER II

AIMS IN ARITHMETIC.

Arithmetic has held a commanding place in the school curriculum for many years and has developed for its justification a series of educational aims. Broadly speaking these may be separated into two groups:

1. Traditional Aims.
2. Practical Aims.

Traditional aims claim for arithmetic all-encompassing virtues. Scholarly articles have been written to show its disciplinary, cultural, aesthetic, and ethical values. The purely disciplinary basis has justified the inclusion of hundreds of socially unnecessary topics, and thousands of archaic and artificial problems. Claim is made that contact with absolute truth aids in setting up higher standards in all kinds of work; that reason, judgment and concentration are trained; that careful analysis is valuable in that it leads one, in considering every problem in life, to exclude the non-essentials and hold rigidly to a definite line of argument. By setting down results with clearness, and stating theories with terseness, mental order is evolved. The cultural value has a certain literary claim. In one's daily reading such expressions as "a changing ratio," "the means and the extremes," "a proportional relationship" etc. do arise, but to justify instruction in whole topics for the sake of literary expressions which should be elucidated in connection with their own

context is absurd. An aesthetic value is claimed. Rythm and orderly arrangement are supposed to inculcate an appreciation of the beautiful; keen pleasure, exalation of mind, a thrill of joy may accompany a student's Q.E.D., but that might well arise from the accomplishment of any completed work. A careful study of arithmetic is claimed to promote ethical standards, though what there is in the study to make one treat his fellow-man better, or love his country more is difficult to say.

That traditional aims dominate courses of study, and that the work mapped out is largely in terms of traditional subject matter is evident in eight out of ten of the courses in use in our city and county schools. Certain California Normal Schools have only recently awakened to the fact that a new era in arithmetic has dawned. The course in use in the Training School of the Los Angeles State Normal School in 1917 contained the following statements:

"The cultural and ethical phases of Arithmetic must be recognized if we are to make our subject of highest educational value. A great responsibility rests upon the teachers of arithmetic. There is constant opportunity for encouraging clear-cut, concise statements of conditions. Laxness and carelessness in speech beget inaccuracy of thought, and a disregard for logical sequence. Poverty of mind, and a patent lack of understanding of data involved, are evidenced by inaccuracy of statement. Clear, direct, coherent and grammatical expression is of vital importance in

Arithmetic.

"Arithmetic, indeed, has a moral worth. It gives the child an opportunity for overcoming obstacles; it develops in him a broadminded, self-sufficient attitude toward difficulties. It disciplines him in persistency in effort toward accomplishment; it serves, in a degree, as a check to flabby inefficiency of thought, and loose vagueness of conclusion. It makes of him an investigator, a questioner, a doubter, a scientist. The feeling of certainty must be his if he is to attain the joy of achievement, and the intellectual delight which comes through mastery.

"Much valuable information is gained through the medium of Arithmetic. Affairs of the business world -- customs, forms, procedures,-- are dealt with and lastingly impressed through practical problems. Hundreds of facts imperative to an understanding of the industrial, commercial and social world are constantly being added to the pupil's knowledge-content without, in any sense, diminishing his opportunity for gaining arithmetical proficiency."

Dr. Frederick Burk of the San Francisco Normal School was among the first of the western school-men to challenge the accepted arithmetical aims, and to declare for a revolution in subject-matter. He proclaimed, characteristically, that there was "something radically rotten in the State of Arithmetic in elementary schools," and recommended that "the pedagogical skeleton in the family closet be dragged forth and its ghost laid."[1]

1. Address: State Teachers' Association. 1910.

While the rank and file of the public schools are dominated by traditional aims, the outlook is hopeful. The movement is away from these principles. Under strong, progressive leaders, in school communities favorable to changes according to social needs, courses are being revised.[1]

The practical aim is the one accepted, and built upon by the advocates of the newer arithmetic. According to this standard arithmetic is a tool to be used in the affairs of everyday life when and where numbering is necessary. Its content must be drawn from industrial and social life and only in so far as it functions in industrial practices and fulfils social needs has it purpose and place in the elementary school program.

The aim in teaching arithmetic is, first, to furnish the pupil the opportunity to further develop his ability in numbering, second, to aid him in acquiring such skill and accuracy in the application of numbering as society demands.[2]

1. Eight Southern California cities: Santa Ana, Pomona, Long Beach, Santa Monica, Pasadena, Riverside, Redlands, San Bernardino, and the Teacher Training Department of the Southern Branch of the University of California (formerly the Los Angeles State Normal School) have banded themselves together in committee, and are building a common course of study founded upon practical aims, and developed along the line of social requirements.
2. This, in effect, is the purpose upon which the course of study of the nine Southern California school-units is built.

Since the aim is practical, the place held by arithmetic in the curriculum can be justified only by the elimination of all those parts which are not useful to society as a whole. Furthermore, since nothing except what is usable can enter into the development of the child's concepts or notions, those parts of arithmetic which are usable by the child or can be made to be of use to him in his interpretations of his surroundings, must be emphasized.[1]

1. Note 2 of page 18.

CHAPTER III

ELIMINATIONS IN ARITHMETIC WHICH HAVE BEEN SUGGESTED OR MADE.

The stirring need for change in the methods of instruction and in the subject matter of arithmetic has been felt in the school world only in the past twenty-five or thirty years. Undoubtedly the most important factor in bringing about this movement toward radical change is the adverse criticism of the school product by the business world. As in all real educational advancements the dynamic force came from the users, the ones who require a practical functioning of theories: the people. Things do not grow well when headed down; the moving force, the germination must come from society; the schools must react to this force.

The practical, as opposed to the disciplinary virtue in arithmetic has now passed the dividing of the ways and is trying to adjust itself to the newer road. Perhaps no subject in the American school system has advanced through more winding ways of experimentation and question in the last two decades than has the subject of arithmetic. Investigators with much enthusiasm, and, we believe, much wisdom, have been making trial of this and that new idea, weighing facts, and establishing truths.

Felix qui potuit rerum cognoscere causas,
we acclaim with Virgil, if that knowledge comes to be his by the pathway of honest doubt. Difference of opinion engenders doubt,

doubt stimulates investigation, and investigation leads to truth. Old forms of thought are passing, old habits of mind give way to the new points of view, and the time honored standards of instruction, whose weakness are closely akin to wickedness, are being entirely displaced and the practical established stoutly in the front line of progress.

In reviewing the literature upon the subject of elimination one finds that which offers much promise. Education is proverbially conservative, and systems are so fixed by devotees that they tend to keep our schools behind the times, but the last years of the nineteenth and the first decades of the twentieth century are hopeful in outlook. By sprinting we may overtake the world.

Arithmetic, in its primary conception, was wholly practical in ideal. The Greeks urged that the common people be instructed in the elements of logistics, the mechanical phase, and Plato mentions its usefulness in trade. Arithmetica, the theoretical aspect, was reserved for the philosopher and the scholar. Arithmetic developed its practical connection because society needed and used it, and retained this as its raison d'etre down to the middle ages, when it was divorced from its utilitarian value and became a subject of pure speculation for philosophers and monks. The disciplinary fanatics dominated for several centuries. The subject became replete with catch phrases and curious problems ingenuously constructed by monks as material for disputation. One needs only open the texts of to-day to find evidences of the per-

sistence of this mediaeval tendency. But through it all, the world needed number and used it. The commercial value of the subject lived because life required it. School and practice were widely severed. The one aristocratically polished minds, the other democratically and doggedly served society. That the vocational phase of our subject must be re-established is clear in view of its history. Many forces, many men and many views of education have combined to make it what it is to-day.

On examining an old treatise upon arithmetic published in England in the sixteenth century a collector of old books tells us of the practical nature of this old English treasure.[1]

The author, indulging in a lament at the ignorance of the people in general which was "pitiful to talk of and most miserable to feele," offers his work which he claims to be excellent to aid all classes, especially the mechanics and soldiery. The study is in the form of a dialogue made up of "gallant speeches full of courtesy." The author begs the authorities "to be acquainted to aid in paying of soldiers' wages, charges of ordinances, powder, shot, munitions, and instruments and what-so-ever falls into business." His problems are typical:

"If a captain over a band of men did set 300 pioneres to worke which on eight houres did cast a trench of 200 rodes, I de-

1. Wingfield; Lewis: A Sixteenth Century Arithmetic. Living Age, 167:315.

mand, how many labourers will be able with a like trench in three houres to entrench a campe of 3400 rodes?" Our present day proportion, the old "rule of three" which was the fetish of our fathers! But for the spelling and phrasing one might well suppose the problem to be taken from our own state text of 1919.

R. B., Schoolmaster, is the author of another old book prefaced 1655, England. It is An Idea of Arithmetick,[1] and contains twenty principles or rules. R.B. defines arithmetic as "The Art of Numbering" (the identical wording of Dr. Ernest Carrol Moore, 1919) and signs himself "Of the Free Schools of Thurlow." The book was compiled for the "furtherance in learning of Sir William Soames' Hopefull Branch, William Soames," Besides the twenty rules it is full of Latinized English, and is built upon the principles of "Ratio or Inequality." Among the rules are:

The Golden Rule, (proportion)
The Rule of Fellowship (partnership)
Alligation
Arithmetical Progression.

The writer is struck by the resemblance between this little book written three and a half centuries past and her own childhood text shelved some twenty-odd years ago. The same rules, the same expressions abound. As William Soames' "Hopefull Branch" was met on the threshold of learning by this pedantry, the writer as a "Hopefull Branch" met and struggled with the same rules, wasting

1. Barnes, Earl: An Old Arithmetic. Academy, 4. p.501.

hours and energy upon those things, dry, dull, deadly and useless drudgery, which bind us by tradition to the past.

Before 1700, we find little record of instruction in arithmetic in the United States, and such as there was was built upon English customs, weights, and measures as the early New England settlers clung to the training of the Old World.

In 1649 Hampton, New Hampshire, employed a schoolmaster "To teach to read, to write, and to cast accounts if it be desired."

In 1653 Dedham, Massachusetts, had a schoolmaster who agreed to teach reading, writing and "the knowledge and art of arithmetic and the rules and practices thereof."

In 1789 arithmetic was required by law in Massachusetts and New Hampshire. There had been a few English texts sparingly used before the Revolution, but the manuscript, the "cyphering book", and rule book became quite common for instruction of boys (girls were exempted from deeper learning) after the act of 1789. The mechanical, rule-of-thumb methods, however, gave them little advantage over the uninstructed girls.

The author of an old textbook published in 1795,[1] makes an impassioned and patriotic plea for the use of United States money, and the elimination of English money and measures in the new Republic. He says, "Let us, I beg you, Fellow-citizens, no longer

1. Gault, R.F.: An old text. Education 20:279.

meanly follow the British intricate mode of reckoning. Let them have their way and us, ours. Their mode is suited to the genius of their government, for it seems the policy of tyrants to keep their accounts in as intricate and perplexing a method as possibl the smaller number of their subjects, then, may be able to estimat their enormous impositions and exactions. But Republican money ought to be simple and adapted to the meanest capacity."

We commend the author as a forward-looking educator, for he further recommends that "Insurance, duties, commissions, indeed, anything reckoned with per cents should be calculated in one head and rule." We, in 1921, are making the same recommendation.

Some recognition of arithmetic as a lower school branch was made by the early colleges.[1] In 1745 Yale required arithmetic for entrance; in 1760 Princeton required candidates "to understand the principal rules of vulgar arithmetic;" and in 1807 Harvard's admission statements required that the entrant "be well instructed in the following rules of arithmetic;" namely, notation, simple and compound addition, subtraction, multiplication and division, together with reduction and the single Rule of Three."

Among the early New England texts[2] which have left an imprint upon our modern books for good and ill is, at least, one book of English authorship. The Schoolmaster's Assistant, 1743, by Thomas Dilworth, was used extensively in this country and held its

1. Brown, E.E: The Making of our Middle Schools. Page 249.
2. Sources: Thomas Dilworth, The Schoolmaster's Assistant. Private library of U.C.Wheet, Los Angeles, California, and U. S. Bulletin no.10. 1917

popularity even after the advent of the comprehensive work of Nicholas Pike.

Dilworth claims to be practical. He has nothing to say regarding the nature and theory of number. His notation and numeration he limits to nine digits. Pike includes seventy-eight digits, duodecillions, dividing his periods, after the English fashion, into six digits each.

Among the topics included in Dilworth's Master's Assistant which educators are still endeavoring to eliminate are:

I. Whole Numbers.

1. The Single Rule of Three direct. (proportion) --- Inverse
2. Compound interest.
3. Simple Fellowship. (partnership)
4. Compound Fellowship. (partnership)
5. Trade Discount.
6. Exchange.
7. The double Rule of Three (compound proportion)
8. Alligation.
9. Progression.

II. Vulgar Fractions.

1. Compound and complex fractions.
2. Reduction of fractions,- Dilworth gives this under twelve different cases, each elaborated, while addition, subtraction, multiplication and division of fractions are accorded three scant pages with no il-

lustrative examples or explanations as to the meaning or use of fractions.

III. Decimal Fractions. (This section includes much subject-matter not included under that head to-day; the need for decimals was not keenly felt until United States money was in general use.)

1. Square root.

2. Cube root.

3. Rule for extracting roots and powers.

4. Reduction: as, Reduce 76 yd. to a decimal of a mile.

5. Applications of Percentage as separated topics: barter, loss and gain, rebate, discounts, interest for years, months, days, purchasing freehold or real estate, annuities.

Among the forty topics treated under decimals some score have already lost place in modern books; nine hold good; and the above group is in question, or has, in part, been eliminated.

IV. Denominate Numbers.

1. Furlong in linear measure.

2. Rood in square measure.

3. Troy weight. (Dilworth's table is similar to the one used to-day.)

4. Circular measure -- similar to the one used to-day.

5. Foreign Money - English Money, in use to-day.

6. Apothecaries weight, in use to-day.

7. League.

8. Hand.

9. Gallons in a barrel, in use to-day.

While very many of the weights and measures given by

Dilworth are eliminated from the present day texts, the writer finds, by comparing, practically all in the texts used twenty-five years ago, and further, finds nearly all in a text by David Eugene Smith, A Grammar School Arithmetic, published in 1904, though intended for reference, chiefly, in the latter book.

A number of other English editions[1] were printed in the United States: Crocker, Wingate, Gough, and the "first purely arithmetical work published in the United States," an edition of Hodder's Arithmetic, Boston, 1719, by J. Franklin.

Arithmetick, Vulgor and Decimal; with the Applications thereof to a variety of Cases in Trade and Commerce was the first American book by an American author, Isaac Greenwood, 1729. This book found small place in the schools and has left little imprint upon our texts for us to commend or condemn.

That Necessary Art made Easy, James Hodder; The Schoolmaster's Assistant, Nathan Daboll; The Scholar's Arithmetic, Daniel Adams, may be mentioned in tracing this early development of the subject in America; but the two outstanding figures, the ones who have exerted the greatest influence upon the development, form, content, and instruction of arithmetic in the United States, the influence of the one of questionable benefit, the other for good, are Nicholas Pike and Warren Colburn.

About 1779 Nicholas Pike of Newburyport, Massachusetts published his first arithmetic, and in 1788 his New and Complete

System of Arithmetic,[1] the first book generally used in the United States, appeared. An examination of this book forces a reluctant admiration for the genius which could conceive, order, and body-forth the content. What shape and aims and agencies of education combined to produce him? What inborn ability invented this divine essence of mathematics? For it must have sprung, Minerva-like, from his fertile brain; practice was no forebear of it.

The first 408 pages are devoted to rules,-- a rule for every page, and sometimes, though rarely, a demonstrated problem to elucidate the rule -- to topics, to problems, to tables, to specific directions of procedure and process; policies of insurance are given eight cases, the Rule of Three recognizes seven complications, while the Inverse Rule of Three and the Double Rule of Three are further sub-divided and distinguished from the direct cases in twelve different sections. There is no overflow or correlation of knowledge threatened here. Everything is safely partitioned and tidily niched. Following the 408 pages devoted to arithmetic are 4 pages of "plain" geometry, 11 pages of "plain" trigonometry, 45 pages of mensuration of specifics and solids in which are introduced practically all the rules of geometry, 33 pages of introduction to algebra designed for the use of academies, and 10 pages of conic sections.

The following table of contents, arranged by Mr. Pike, himself, gives a notion of the comprehensiveness of the work.

1. Pike, Nicholas: New and Complete System of Arithmetic. Reprint, private library of H.C. Wheat, Los Angeles, California.

Table of Contents of A New And Complete System of Arithmetic.

By Nicholas Pike.

This table is inadequate to give the full significance of the content, however. To illustrate: Under the head "Rules for reducing Federal coin and currencies of the several United States, also English, Irish, Canada, Nova Scotia, Livres, Tournois, and Spanish milled Dollars each to the par of the other" Mr. Pike gives 78 rules for the exchange of coin among the States alone, and then treats the foreign nations with equal consideration.

As a general reference book, or, by selection, a text for special technical schools the book was undoubtedly valuable. Some of the rules are quite unintelligible to modern students. Under Trett and Tare, Pike says: "Deduct the tare and trett, divide the suttle by 168 and the quotient will be the cloff, which subtract

from the suttle and the remainder will be the neat." Evidently, "trett," "tare," "suttle," "cloff" and "neat" were of the vocabulary of the day, and this rule for the weighing and handling of merchandise was useful to a group of traders. One can hardly conceive of these rules being useful to the average citizen in his daily life, and even the advanced students, for whom the text was intended, could have found little with which their experience corresponded. Yet, shall we criticize Mr. Pike's work while our own texts retain surveyor's measure and apothecaries weight?

Benjamin West, in a criticism of the book says, "The volume contains, besides what is useful and necessary in the common affairs of life, a great fund of amusement and entertainment. The mechanic will find in it much that he may have occasion for; the lawyer, merchant and mathematician will find an ample field for exercising their genius."

This book was the gem puzzle of its day. Many pleasing and diverting questions are included in its lists of problems. These problems are connected with questions as to the number of changes which can be rung on a chime, how many different positions a person can assume at a dinner party, how many variations can be made of the alphabet, and many others which reflect the influence of mediaeval disputation. We find the forbear of our puzzle of the fox, the geese and the bags of corn and the complications arising from carrying them across the river two by two, which touches a responsive association in the minds of the students of

the early nineties. Then, in geometrical progression, is that apparently foolish bargain of the merchant who sold 39 yards of fine velvet for 2 pins for the first yard, 6 pins for the second yard, 18 pins for the third, etc. which turns out most profitable to the seller after all. One of our fairly modern books has a problem wherein a crafty person offers a hundred acres of land at two pennies for the first acre, 6 for the second, etc. which the writer has used recently as a diversion to the delight and wonderment of her pupils.

Pike's text ran through five editions, the fifth printed in 1832, and despite the reaction created by Warren Colburn's First Lessons, is the book which is most responsible for the necessity for extensive eliminations to-day.

The great reformer in American arithmetic was Warren Colburn. He made a protest, and a vigorous one, against the deep-rooted evils of the mediaeval and English notion of teaching number as symbols, and of clinging to definisions and rules. He endeavored to seek out and seize upon the instincts of the child and use these as a factor in his educatin. On reading his masterly address delivered before the American Institute of Instruction in Boston in 1830 on Teaching of Arithmetic[1], one realizes he possessed

1. Reprinted in Elementary School Teacher, v. 12, (1911-12) pages 463-480.

an insight into the educative process far ahead of his time, and which we are but now coming to realize.

Colburn was avowedly a follower of Pestalozzi. In his preface to the second edition of his First Lessons is a lengthy tribute to Pestalozzi which gives clear evidence of his acquaintance of the Pestalozzian plan, although he was largely independent in his conceptions and in the carrying out of his theories. Mr. Thomas Sherwin, principal of the high school, Boston, (1830) says: "I regard Mr. Colburn as the great benefactor of his age with respect to the proper development of the mathematical powers. Pestalozzi, indeed, first conceived the plan; but Mr. Colburn realized the plan; popularized it, and rendered it capable of being applied to the humblest mediocracy. Indeed, I regard First Lessons as the ne plus ultra of primary arithmetic."[1]

First Lessons in Intellectual Arithmetic,[2] published first in 1821 achieved almost incredible success as a text. It was followed by The Sequel to First Lessons[3] a year later. First Lessons is really a "Minimum Essentials" of arithmetic. Mr. Colburn's idea was to eliminate all useless material, to teach by the use of concrete objects, and to connect each question with the activity of the child himself.

1. Elementary School Teacher, vol. 12, page 424.
2. University of California Library - reprint.
3. University of California Library (original, edition of 1834)

A comparison of the table of contents of First Lessons with that of Nicholas Pike's New and Complete System of Arithmetic is illuminating.

Table of Contents of Warren Colburn's First Lessons.[1]

Part I.

Sec. I. Addition and subtraction.
Sec. II. Multiplication.
Sec. III. Division. - Idea of fraction introduced.
Sec. IV. Fractions; multiplication of an integer by a fraction.
Sec. V. Principle of fractions applied to larger numbers.
Sec. VI. Division of an integer by a fraction.
Sec. VII. Combinations of preceding and multiplication table from 10 x 10 up to 10 x 20.
Sec. VIII. Reduction of fractions to higher terms and integers to fractions.
Sec. IX. Multiplication of a fraction by an integer.
Sec. X. Mostly drill on Sections IV and IX.
Sec. XI. Division of one fraction by another.
Sec. XII. Fractions written in fractional form.
Sec. XIII. Reduction of fractions to a common denominator; addition and subtraction of fractions.
Sec. XIV. Division of fractions by integers and multiplication of a fraction by a fraction.
Sec. XV. Division of integers by fractions and a fraction by a fraction.

Colburn specifically mentions the elimination of The Rule of Three (proportion) of Cube Root, and of Square Root, and suggests that Denominate Numbers, Percentage, Interest and Mensuration should not be taken as bases for separate chapters or even distinct topics.

1. Colburn does not give a table of contents in the First Lessons, and this has been made from a study of the material given in the various sections.

It is an interesting fact to note that in 1821 Warren Colburn recommended the elimination of Square Root from the elementary school text and that the writer, after a lapse of one hundred years, should find it necessary to make the same recommendation.

Following Colburn came the books of Joseph Ray, books built along the line of Pile rather than Colburn, and from the waning of Colburn's influence to the latter part of the nineteenth century, arithmetic was taught as a mental discipline. The body of matter accumulated, tradition retaining what had gone before, and a desire for new disciplinary material adding to the bulk.

Throughout this static period people accepted rather than investigated. Opinion served for information and that opinion was generally second-hand. It is related of a learned judge that he once praised a retiring witness in these words, "You are entitled to great credit, sir. You must have taken infinite pains with yourself. No man could naturally be so stupid." Our great-parents' arithmetic courses show a like effort.

While there was no concerted action toward simplifying and relating numbering to use during these middle years, one finds protests against the conventional teaching and the clinging to the dogma of formal discipline. A number of writers make timid pleas that schools avoid more empirical rules and keep in view the practical and commercial value of arithmetic as an art of computation. They assert that pupils trained to operate according to trade-rules which they do not understand are quite at loss to help

themselves in new situations.[1] "But, although, by this method (disciplinary) the scholar may be well prepared for many computations which he will have occasion for in practical life, yet he will be quite at a loss how to help himself whenever a case shall come up to which he cannot apply his rule exactly as he has learned to use it."[2]

Little further was done in the way of simplifying arithmetic until 1887 when President Francis A. Walker attacked the problem in the Boston schools, and his investigations resulted in a recommendation by the school committee that the following subjects be eliminated from the course of study:

1. Mensuration of unusual surfaces and solids.
2. Compound proportion.
3. Compound interest.
4. Equation of payments.
5. Exchange.
6. Metric System.
7. Compound partnership.[3]

With the beginning of the twentieth century, significant reforms in Arithmetic were advocated by certain noted educators throughout the East and the Middle West. It was their purpose to search out the practical in arithmetic and to omit all other material. In 1903 C. W. Stone sent out a questionnaire

1. Peacock, G.: Educational Value of Arithmetic. London Quarterly, 1858.
2. Barnard's Journal of Education. vol.8 (1860) page 170.
3. Jessup, Walter A.: Educational Research, School and Society. July 24, 1915. Page 137.

to business men of Indianapolis, requesting their opinions on the utilitarian value of the work given in arithmetic. The replies indicated that certain topics had absolutely no use in the business world.

Dr. Frank McMurry in an address on "What omissions are advisable in the present course of study, and what should be the basis for the same", delivered in 1904 before the National Department of Superintendence, recommended that the following topics be eliminated from arithmetic:

1. Apothecaries Weight.
2. Troy Weight.
3. Examples in Longitude and Time, except the very simplest.
4. The furlong in linear measure.
5. The rood in square measure.
6. The dram and the quarter in avoirdupois weight.
7. The surveyor's table.
8. Table on folding of paper.
9. All problems in reduction, ascending and descending, involving more than two steps.
10. The G. C. D. as a separate topic, but not practice in detecting divisibility by 2, 3, 5, and 10.
11. All common fractions except those of a very low denomination and customary in business.
12. All work with the L. C. M. except of such very low denominations as those just mentioned.
13. Complex and compound fractions as separate topic.
14. Compound proportion.
15. Percentage as a separate topic, with its cases.
16. True discount.
17. Most problems in compound interest, and all in annual interest.
18. Problems in partial payments except those of a very simple kind.
19. The same for commission and brokerage; for example, all problems involving fractions of shares.
20. Profit and Loss as a special topic.
21. Equation of payments - made unnecessary by improved banking facilities.

22. Partnership - made unnecessary, in the old sense, by stock companies.
23. Cube Root.
24. All algebra, except such simple use of the equation as is directly helpful in Arithmetic and in other subjects in the shcool life of the pupil.

In addition to all of these, arithmetic may be omitted as a separate study throughout the first year of school on the ground that there is no need of it if the number incidentally called for in other work is properly attended to.[1]

In 1904 Joseph V. Collins of the State Normal School of Stevens Point, Wisconsin made some very definite recommendations regarding eliminations. In a discussion, "The Superintendent and the Course of Study" he says, "If instruction in schools in arithmetic, is to be brought up to a place where it will be respected by the business world it will have to include only operations called for in business transactions."

He suggests that superintendents go through the text books they may have in use with a big blue pencil and see to it that the following things are omitted:

1. The C. C. D. as a special topic.
2. The L. C. M. (he suggests it to be put with addition of fractions).
3. Longitude and time (put with geography).
4. Taxes and duties (put with civil government).
5. Problems involving large numbers.
6. A vast deal of matter commonly given in denominate numbers.

He says, "All of addition, subtraction, multiplication, and division of compound denominate numbers should be marked de-

1. McMurry, Frank: Addresses and Proceedings of the National Educational Association, 1904 pages 194-202.

lete, as also all problems which involve quantities expressed in more than two denominations. Such a problem as: "Reduce 2mi. 3rd. 5yd. 2ft. 5 in. to inches" is as absurd as Munchausen history. It is evidently the product of some schoolmaster's disordered invention. Such, however, is the force of custom that numerous problems of this type are to be found in most of the arithmetics of this day.[1] The superintendent should draw his blue pencil through all the tables of denominate numbers except avoirdupois weight, linear, square, cubic, dry, liquid, and time measures, and all problems under them." He further speaks of typical problems in analysis and proportion being, "strictly speaking, not practical."

Mr. Collins would include simple algebra and geometry in the upper elementary grades. "Let geometry be substituted for mensuration, and let the puzzle problem give place to algebra."[2]

In 1909 G. M. Wilson made a study of the social and business use of arithmetic, and published his findings in the Sixteenth Year Book of the National Society for the Study of Education. He says:

"In connection with the working out of a course of study in Arithmetic at Connersville, Indiana, a few years ago, an attempt was made to get the judgment of the business community on a number of arithmetic topics. As a result of this cooperation, the

1. The writer adds, "And this day." Note the California State Text of 1919.
2. Collins, Joseph: The Superintendents and the Course of Study. School Review, 27. Pages 83-89.

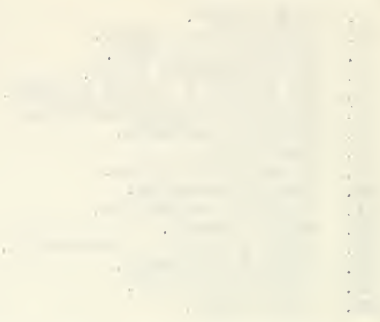
business men of the city voted to omit the following topics from the arithmetic courses:

1. Troy Weight.
2. Apothecaries Weight.
3. Longitude and Time.
4. The surveyor's table.
5. The Greatest Common Divisor.
6. The Least Common Multiple.
7. Complex Fractions.
8. Cube Root.
9. Compound Fractions.
10. Foreign Exchange.
11. Compound Proportion.
12. True Discount.
13. Cases 2 and 3 in Percentage.
14. Compound Interest.
15. Partial Payments.
16. Partnership. [1]

In 1910 the Baltimore School Commission criticised the time expenditure and topical emphasis in arithmetic and suggested alterations, adjustments and eliminations be made to meet the needs of the business world[2] and in 1919 recommended these eliminations:

1. G. C. D. and L. C. M. of large numbers otherwise than by factoring.
2. Fractions with large and unusual denominators.
3. Complex and Compound fractions.
4. All measures not actually in use in the community at large: troy, apothecaries, dram.
5. Reduction of decimals to common fractions, and decimals beyond thousandths should receive little emphasis.
6. Circulating decimals. The topic should be studied as a part of infinite series in algebra.
7. Square root and cube root except by factoring.
8. Profit and loss as a separate topic.
9. True discount. Bank discount has taken its place entirely.

1. Wilson, G.M.: Sixteenth Yearbook of the National Society for the study of Education. Chapter VIII, Page 128.
2. Bureau of Education Bulletin 1911, No. 4, page 76.



10. Partial payments in the form of state rules and irregular indorsements.
11. Equation of payments.
12. Compound proportion has been largely replaced by unitary analysis. Simple proportion is of some importance but is best treated as an equality of two fractions.
13. Business problems which do not conform to the usage of the day.
14. Large numbers and exercises involving many numbers should also be excluded as a rule. [1]

In 1911 the American Committee No. 1 of the International Committee on the Teaching of Mathematics reported on Mathematics in the Elementary Schools. This Committee, through its investigations, found that school and business people felt an urgent need for simplifying courses of study:

1. By using small numbers in work in arithmetic.
2. By eliminating topics that are unduly confusing.
3. By eliminating obsolete problems, topics and processes. [2]

In 1912 or 1913 Dr. Walter A. Jessup and Dr. L. D. Coffman determined to submit the problems of elimination and enrichment of the curricula to the superintendents of all cities of the United States with a population of 4,000 or over. This investigation, as reported in the Fourteenth Year Book of the National Society for the Study of Education, indicates a decided tendency among the superintendents of these cities to accept the general proposition of either eliminating or giving less attention

1. Course of Study - Baltimore County. 1919. Pages 312-313.
2. Jessup, Walter A.: School and Society, 2. Page 137.

to the topics originally suggested by Dr. McMurry. They recommended that these be added to, or changed, by further study. The reports show, also, a strong tendency among the superintendents in favor of more attention to economic and business applications of arithmetic. From this return, Dr. Jessup recommended that an economy of time might be effected by the elimination of the following topics from the elementary course of study:

1. Apothecaries Weight.
2. Alligation.
3. Aliquot Parts.
4. Annual Interest.
5. Cube Root.
6. Cases in Percentage.
7. Compound and Complex Fractions of more than two digits.
8. Compound proportion.
9. Dram.
10. Foreign Money.
11. Folding Paper.
12. The long method of C. C. D.
13. Longitude and Time.
14. Least Common Multiple.
15. Metric System.
16. Progression.
17. Quarter in Avoirdupois.
18. Reduction of more than two steps.
19. Troy Weight.
20. True Discount.
21. Unreal Fractions. [1]
(Chart, p.119 - Fourteenth Year Book.)

Recent writers on the teaching of Arithmetic suggest many eliminations. In Brown and Coffman's How to Teach Arithmetic, published in 1914, the following eliminations are recommended:

1. Jessup, W. A. and Coffman, L. D.: The Supervision of Arithmetic. Chapter 1. Pages 8-13.

1. G. C. D.
2. L. C. M.
3. All obsolete tables in denominate numbers and all tables that are of use to the specialist only.
4. Long or unnecessary reductions.
5. Circulating decimals.
6. All applications of percentage that do not conform to present-day practices.
7. True Discount.
8. Equation of Payments.
9. Cube Root.
10. Progression.
11. Compound Proportion.
12. Problems which require long and involved solutions.
13. All fractions except those used in every-day business life -
    a. Long fractions.
    b. Complex fractions.
14. Partial payments.
15. All topics which time or changed social conditions have rendered obsolete.[1]

In 1915 the Committee on Eliminations of the Iowa State Teachers' Association recommended a sweeping elimination of obsolete and useless topics and materials from the common branches. A second report of a more definite nature was made in 1916 in which this Committee recommended that the following eliminations be made in arithmetic:

1. Long method of greatest common divisor.
2. Most of lowest common multiple.
3. Long, confusing problems in common fractions.
4. Long method of division of fractions. (Always invert and multiply.)
5. Complex and compound fractions.
6. Apothecaries' weight, troy weight, the furlong in long measure, the rood in square measure, dram and quarter in avoirdupois weight, the surveyor's table, the table of folding paper, tables of foreign money, all reduction of more than two steps.

1. Brown, Joseph C. and Coffman, Lotus D. How to Teach Arithmetic. Pages 117-118.

7. Most of longitude and time.
8. Cases in percentage. (Make one case by using x and equation.)
9. True discount.
10. Most of compound and annual interest.
11. Partial payment, except the simplest.
12. Profit and loss as a separate topic.
13. Partnership.
14. Cube root. [1]

David Eugene Smith in a chapter on arithmetic in Teaching Elementary School Subjects by L. W. Rapeer and others recommends that relative values in arithmetic be considered. He would eliminate:

1. Fractions with large denominators.
2. Division of a fraction by a fraction.
3. Multiplication of a fraction by a fraction.
4. Cases in percentage.
5. Subject matter found of little value in the business world:
   - Square root.
   - Cube root.
   - Progression.
   - Equation of payments.
   - Proportion. [2]

The Teaching of Arithmetic published by John Charles Stone in 1918 recommends the elimination of:

1. Greatest Common Divisor.
2. Addition and subtraction of fractions with large or unusual denominators.
3. Least Common Multiple.
4. The more complex forms of complex fractions.
5. Obsolete tables and those used in specialized vocations.

1. Wilson, G. M.: Report of the Committee on Elimination of Subject-matter to the Iowa State Teachers Association.
2. Smith, David Eugene: Teaching Elementary School Subjects. Rapeer and Others. Pages 207-249.

6. Impractical reductions in denominate numbers.
7. Addition, subtraction, multiplication, and division of compound denominate numbers.
8. The present type of inverse problems in fractions and percentage.
9. The various short methods of finding interest.
10. All inverse problems of interest.
11. Partial payments.
12. Annual interest.
13. Undue emphasis upon the discounting of interest-bearing notes.
14. True discount.
15. Partnership.
16. Proportion as a general method of solving problems.
17. Foreign and domestic exchange.
18. The measurement of uncommon areas and volumes.
19. Square root and the Pythagorean Theorem.
20. The metric system.[1]

This study seems to justify the conclusion that up to the twentieth century tradition tended to keep subject matter which had once been added; that new subject matter was much more rapidly incorporated than obsolete matter discarded. This resulted in a mass of material which was cumbersome, impractical, much of it utterly useless. But with the beginning of the twentieth century, certain unprejudiced searchers after truth, realizing the necessity for radical change, began a vigorous campaign for the elimination of this useless material picked up in the development and progress of the subject. These investigators claimed the aim of arithmetic is to enable one to do intelligently the common work of the world, and they proceeded to discover from the world what portions of the subject are serviceable in interpreting

1. Stone, John Charles: The Teaching of Arithmetic. 1918.
Note: The Stone-Millis texts are faithful to this ideal.

our surroundings from a number point of view, and to eliminate the rest.

Many forces, many men, and a commendable amount of scientific investigation have combined to make the subject what it is to-day. What we have achieved thus far has served but to increase our interest, and to urge us to further investigations of every phase of arithmetical work.

CHAPTER IV

MINIMUM ESSENTIALS IN ARITHMETIC.

"The minimum essential in arithmetic is the ability on the part of the individual to do practical calculations such as are needed by the average citizen in his daily life."[1]

Socrates "cursed as impious him who first separated the just from the useful," and we add, "him who first separated common sense and arithmetic."

During the past two decades forward-looking teachers and school administrators have been concerning themselves with the question, "What is essential?" and have been laboring to relieve the commonly accepted curriculum of its unjust burdens. Public opinion itself is making a strong appeal that the arithmetic taught be more in tune with life. In the expression of dissatisfied business men there is very little suggestion of details in the solution of the problem, but there is a stern demand for results. Schools have been tardy in response, but leaders have been studying to determine what parts of arithmetic have definite utility, and are providing special opportunities for giving exclusive attention to those aspects of them.

President Francis A. Walker of the Massachusetts Institute of Technology began the crusade in 1887. He declared

1. Report of Committee on Course of Study in Arithmetic. Los Angeles State Normal School. 1919.

that "a false arithmetic has grown up which has largely crowded out the place of true arithmetic --- The most jagged fractions such as would hardly ever be found in actual business operation, e.g., 11/29, or 13/27 are piled one on top of another, to produce an unreal and impossible difficulty; the child having been furnished with such an arithmetical monstrosity, is set to multiplying or dividing it by another 'compound and complex fraction' as unreal and ridiculous as itself. All this sort of thing in the teaching of young children is either useless or mischievous. The charge I make against the existing course of study is that it is largely made up of exercises which are not exercises in arithmetic at all, or principally, but are exercises in logic, and secondly, that as exercises in logic, these are useless or mischievous --- Generally, if not universally speaking, whatever in education is hard, is wrong."[1] More than thirty years have gone by since then, and the arithmetic taught is still, for the most part, the traditional thing which Dr. Walker found so meaningless. We use a year or more to teach fractions. Let any individual watch himself carefully for a month and discover when he had occasion to add 2/5 and 3/4, or subtract 2/5 from 3/4, or to multiply a fraction by a fraction. I challenge anyone to discover an occasion in life for dividing a fraction by a fraction.[2] Yet in practical-

1. Walker, Francis A: Arithmetic in the Boston Schools, Academy (Syracuse) 1887. Page 433.
2. An Education Seminar group composed of school superintendents, principals and teachers of experience accepted this challenge, but failed, during a six weeks course, to find le itimate oc-

ly any text book used in our land, we find this sort of problem: 3/29 x 11/19, 16 5/23 ÷ 13/43.

Too often, even now, children are worried over the number of cubic inches in a gallon, compound proportion, cube root, aliquot parts, such problems as, "If the principal is $643.87, the time 5 years 7 months and 25 days, and the amount $872, what is the rate?" This insane query is from the writer's childhood text: "How many gills in 2 hogsheads, 31 gallons, 3 quarts, and 2 pints?" A liquor dealer, retailing in gill quantities, may have found occasion for such computations, but we could hardly class this an essential since the Volstead Act. From the addition, subtraction and multiplication problems propounded, one would suppose our children were all destined to become millionaires. There are rarely any calculations which involve less than a hundred thousand.

Following Dr. Walker's lead, Albert G. Boyden of the Bridgewater, Massachusetts, Normal School, in 1894 revised his course of study and the methods of instruction. He says, "Arithmetic is too often taken in a merely mechanical way, the pupils working by rule with much cyphering and little thinking --- The study of arithmetic must be enriched --- our pupils should get it in less time than is usually given to the subject, and get the power to think for themselves. How shall this be accomplished? By better teaching, by better arrangements of subjects --- by use of smaller numbers, less figure work, by making and solving such

problems as occur in actual life."[1]

In 1902, President William R. Harper of the University of Chicago proposed a scheme for saving two years' time in the completion of a college course. His suggestion was a six-year elementary course. Dr. John Dewey, in discussing this problem says, "The proper aim of elementary tuition is to organize the instincts and impulses of children into working interests and tools --- Six years ought to be enough to accomplish this task."[2] Out of this effort to save time grew the far more vital question, "What is worth knowing?"

Dr. Frank McMurry[3] delivered an address on arithmetic before the Department of Superintendence of the National Educational Association in 1904 which stirred educators to further interest in essentials. He says the content of studies should be determined, first, by social needs, second, by the child's ability to comprehend. He decries the "harmonious development of the faculties" theory and the puzzle problem that puzzles the teacher. He says, "Life is too full of large specific aims to be attained to allow for work that has no really tangible object." He rejects:

1. Boyden, Albert C.: Education 14: page 390. March 1894.
2. Dewey, John: School Review. January 1903.
3. McMurry, Frank M.: What Omissions are Advisable in the present Course of Study and What should be the Basis for the Same? National Educational Association Report. Page 194. 1904.

1. Whatever cannot be shown to have a plain relation to some real need of life...
2. Whatever is not reasonably within the child's comprehension.
3. Whatever is unlikely to appeal to his interests...
4. Whatever topics and details are so isolated or irrelevant that they fail to be a part of any series or chain of ideas...

These principles Dr. McMurry pointedly applied to arithmetic, and his work became the basis for much investigation and study.

Guy M. Wilson and his teachers at Connersville, Indiana in 1909, obtained from the business community an opinion on enrichments and inclusions in arithmetic.

The business community, through their merchants, bankers and factory superintendents, expressed themselves in favor of more attention in the public schools to the following topics:

1. Saving and loaning money.
2. Mortgages.
3. Modern banking methods.
4. Building and loan associations.
5. Keeping simple accounts.
6. Investing money.
7. Bonds as investments.
8. Real estate as investments.
9. Marks of a good investment. (It is estimated that the get-rich-quick concerns fleece the American people out of $60,000,000 a year.)
10. Taxes, levies, public expenditures.

11. Profits in different lines of business.
12. Life insurance as protection and investment.[1]

In 1911, a course of study based upon these findings was issued. This Connersville Course became the subject of study and criticism by Dr. Jessup in the University of Iowa, and by Dr. Coffman at the University of Illinois. They conceived the idea of continuing the study of inclusions and enrichment through the superintendents of all cities of the United States with a population of 4,000 and over.

Through the reports made by the school superintendents, Dr. Jessup recommended that more attention be given to the following topics:

1. Addition.
2. Subtraction.
3. Multiplication.
4. Division of whole numbers and fractions.
5. Saving money.
6. Public utilities.
7. Public expenditures.
8. Insurance.
9. Taxes.
10. Percentage.
11. Profit.
12. Building and Loan.
13. Investments.
14. Interest.
15. Banking.
16. Borrowing.

1. Wilson, G. M.: Survey of Social and Business Use of Arithmetic, Sixteenth Yearbook, page 128. (Also in 1916 reportof Committee on Elimination of Subject Matter, Iowa State Teachers Association, and in A Survey of the Social and Business Usage of Arithmetic, Ph.D. Thesis, 1919.)

17. Levies.
18. Stocks and Bonds.[1]

These same superintendents, Dr. Jessup states, expressed themselves as overwhelmingly in favor of giving special attention to the fundamentals of addition, subtraction, multiplication, division, and to fractions.

In 1915-1916 Professor Walter S. Monroe made an investigation of the economy of time in arithmetic. Taking as the aim in the teaching of arithmetic the equipping of the pupil (1) with the knowledge of facts, principles, and relationships existing between quantities in the solution of practical problems, and (2) with the skills which are necessary to perform these operations, he endeavored to determine what problems are practical.

His major purpose in this study was to secure lists of arithmetical problems which arise in human activities, and which possess that degree of utilitarian or socializing value which justifies their being designated as minimal essentials of purpose. These he groups under:

1. Occupational activities.
2. Activities of the home.
3. Personal activities.
4. Activities of school children.[2]

Dr. Calvin N. Kendall and Dr. George A. Mirick, in 1915,

1. Jessup and Coffman: Supervision of Arithmetic, Attitude of Superintendents, page 15. (Also Fourteenth Yearbook.)
2. Monroe, Walter S.: Economy of Time in Arithmetic. Sixteenth Yearbook, page 111.

after a commendable narrowing of the bounds of study by eliminating a mass of useless material, established the following as the legitimate field of elementary mathematics.[1]

I. Counting numbers.

II. Reading numbers.
1. Integers - Arabic and Roman.
2. Common Fractions.
3. Decimal Fractions.
4. Denominate Numbers.

III. Writing numbers.
1. Integers - Arabic and Roman.
2. Common Fractions.
3. Decimal Fractions.
4. Denominate Numbers.

IV. The Processes.
1. Addition.
2. Subtraction.
3. Multiplication of
4. Division.
(a) Integers.
(b) Common Fractions.
(c) Decimal Fraction to three places.

V. Percentage applications.
1. Trade or Commercial Discount.
2. Profit or Loss.
3. Commission.
4. Simple Interest.

VI. The following subjects should be treated largely for information purposes:
1. Taxes.
2. Insurance.
3. Stocks.
4. Bonds.
5. Bank Discount.
6. Compound Interest.

VII. Denominate Numbers in useful problems of community value.

1. Kendall and Mirick: How to Teach the Fundamental Subjects. 1915.

VIII. Geometry in so far as it is required in mensuration, and in making and reading working drawings in shop work.

IX. Algebra in so far as the use of letters is required in simple formulas in mensuration and in simple problems solved by the equation method.

Minimum Essentials by David Eugene Smith, as found in a chapter on arithmetic in Teaching Elementary School Subjects, by Rapeer and others are as follows:[1]

(Work for the first, second, third, and fourth grades is by Miss Worden.)

1st Grade: (1) Count to 100 by 1's, 2's, 5's, 10's. (2) The simple combinations to 10 or 12. (3) Roman numerals as seen on clock face. (4) 1/2, 1/4 in concrete way. (5) Foot, inch, yard.

2nd Grade: (1) 1,000 - reading, writing. (2) Counting by 2's, 3's, 4's, 9's, 10's. (3) Remainder of 45 combinations. (4) Coin should be recognized - $, ¢. (5) 1/2, 1/3, 1/4, 1/8 applied. (6) Multiplication tables to about 5 x 10. (7) Addition of two-figure numbers not involving "carrying", and the subtraction of such numbers. (8) The square and circle.

3rd Grade: (1) Multiplication tables completed. (2) Separate numbers into their prime factors and learn the simple factors. (3) Division by one digit, using long division in the latter part of the year. (4) Problem interpretation.

4th Grade: (1) Linear measure. (2) Volume. (3) Addition and subtraction of simple fractions. (4) Simple decimals.

5th Grade: (Recommended by David Eugene Smith) (1) Review four fundamentals with whole numbers. (2) Continue simple fractions, stressing multiplication of fractions. (3) Compound numbers - "Happily this is becoming less prominent." (4) Decimal fractions are usually taken up.

6th Grade: (1) Decimal fractions. (2) The elements of percentage.

1. L.M. Rapeer and Others: Teaching Elementary School Subjects. 1917. Chapters 9 and 10.

7th Grade: (1) The work of this grade is civics, economics, or sociology, not mathematics. (2) Interest (some mathematics). (3) In other countries: a. Intuitional geometry. b. Simple linear equation in one unknown. c. Graphs. d. Factoring. ("We may hope for this")

8th Grade: (1) Same as 7th. (2) Dramatize the civics.

Summary of Minimum Essentials
by David Eugene Smith.

1. Addition)
2. Subtraction) Whole Numbers
3. Multiplication)
4. Division)

5. Addition) Of decimal fractions as shown
6. Subtraction) in the case of U.S. money.

7. The ability to find a fractional part of a number.
8. Finding of some percent of number.
9. How to multiply and divide a mixed number ($, ¢) by a whole number.

George Herbert Betts in his Class-Room Method and Management, published in 1917, says, "The main purpose in arithmetic is concrete, direct, practical, applied. It is the business of Arithmetic to enable one to do the ordinary numbering and computing required in the common economic and social relations. The knowledge required should be:[1]

1. How to count objects of all kinds. How to count by naming numbers only. How to count by twos, threes, etc.

2. How to read and write numbers of ten to twelve figures.

3. The tables and processes involved in addition, subtraction, multiplication and division of whole numbers.

1. Betts, George Herbert: Classroom Methods and Management. 1917. Pages 218-219.

4. Common fractions, and their addition, subtraction, multiplication, and division with the use of such denominators as are commonly used in business. A similar knowledge of decimals involving up to three places.

5. The common tables and measures employed in the ordinary life routine of the average man or woman. These are: measures of length, angle, surface, volume and capacity, quantity, weight, time, money, value.

6. Our monetary system, denominations, and the various business practises involving the use of checks, drafts, notes, mortgages, etc.

7. Percentage, and its simpler applications to practical business uses.

8. Simple mensuration, applied to lines, angles, surfaces, volumes.

Attitudes to be developed:

1. A tendency not to be satisfied with guessing or approximation, but to insist on finding out through the use of figures on all essential matters involving numerical values.

2. Standards of business accuracy that will result in the keeping of an accurate account of all personal or household receipts and expenditures. This will make possible a proper adjustment of expenditure to income, and also a right balance among the different objects for which money is spent.

3. Unwillingness to rely on general estimates or rough approximations with reference to projects planned, as improving a home, or a farm, taking a trip, investing in an automobile, etc.

4. Insistence on detailed and accurately kept records of profits or losses from the different enterprises of farm, shop or business.

5. The development of such a sense of values and the inevitable logic of figures as will render one proof against the get-rich-quick schemes planned by unscrupulous promoters to catch those who through ignorance of business believe wealth to be attained by some kind of magic.

6. A sense of pleasure and satisfaction in the use of figures and in the certainty which comes from their wise application

to one's affairs.

John Charles Stone in his Teaching of Arithmetic, 1918, makes the aim of arithmetic practical, and outlines the essentials as follows:

1. Efficiency in computation.

2. A social insight into business and industrial practices that will enable one to interpret references to such practices met in general reading or in social and business intercourse.

3. Power to express and to interpret the numerical expressions of the quantitative relations that come within our experiences.

4. The habit of seeing such relations, particularly those that are vital to our welfare.[1]

Dr. Junius P. Meriam of the University of Missouri, perhaps more than any other educator, has stamped the traditional arithmetic as non-essential. He says that the best way to teach arithmetic is not to teach it at all; that there should be no regular class periods and no regular texts; that arithmetic is a school subject presented to keep the child occupied, to keep him from worse behavior, without considering the outcome of the occupation; that the only arithmetic worth bothering with is that which the child comes face to face with in life; that, however, boys and girls should, and will, have considerable to do with arithmetic as they experience quantities and measurements as they help to do those things about them, and through these will learn to handle arithmetic processes. He further recommends that we

1. Stone, John C.: Teaching of Arithmetic. 1918.

construct a school course in terms of normal activities.[1]

In his Child Life and the Curricula[2] Dr. Meriam criticizes unfavorably the courses of study in general use, all textbooks in part or in whole, and the methods of presentation. He says: Arithmetic is a cross section of a great variety of experiences in the quantitative level."[3] "Arithmetical abilities can be measured by following pupils into stores, shops, factories and other places of employment and there taking into account the arithmetical calculations made as part of their work."[4] "School arithmetic is strictly a form subject. It has not yet approached the study of quantitative aspects of our invironments and our real adjustments."[5]

Dr. Meriam's attitude toward arithmetic in the elementary school curriculum reminds one of that justly famous treatise upon "Snakes in Ireland", the preface of which contains the incidental observation that there are no snakes in Ireland.

Dr. Edward Lee Thorndike states his attitude toward essentials in arithmetic in this letter:

"You will find my opinion concerning what should be in and what should be left out of the elementary school course in mathematics worked out fully in the Thorndike Arithmetic. Every-

1. Personal interview with the writer, January, 1921.
2. Meriam, J.P.: Child Life and the Curriculum. 1920.
3. Ibid page 419.
4. Ibid page 467.
5. Ibid page 286.

thing in those except the few exercises marked "optional" up to page 248 of Book III should, in my opinion, be left in, together with a selection from the material on pages 249 to 286, as stated on page 249.

Yours truly,

E. L. Thorndike."[1]

In Thorndike's newest book, New Methods in Arithmetic, 1921, a book built entirely upon the textbook material in the three volumes of the Thorndike Arithmetics[2] (now adopted as the State Text in California) he advocates useful computations as opposed to indiscriminate ones, facility and absolute accuracy with small numbers; genuine problems; arithmetic for life. He urges that the older methods be discarded and that the newer arithmetic, founded upon common sense, common requirements, common needs be given trial, and is of the belief that this will win assent and confidence on merit.

From this examination of the works of these recognized leaders in thought, one seems justified in the conclusion that the modern aim in arithmetic is a practical one, and that through research, teachers, superintendents, and recognized educators are endeavoring to find out what are the clear-cut first essentials, and what shall constitute a minimal course of study in arithmetic.

1. Extract from personal letter received by Chairman of Committee on Minimum Essentials in Arithmetic, Southern Branch University of California. 1920.
2. See appendix.

PART TWO

THE INVESTIGATION

CHAPTER I

SOURCES AND METHODS OF COLLECTING THE DATA.

The initial impetus for this investigation was the common work of a Committee on Minimum Essentials in Arithmetic of which the writer is secretary.[1] This committee was appointed and began its work on February 9, 1918. A group of Superintendents of Southern California City Schools,[2] and the President of the Los Angeles State Normal School (now the Southern Branch of the University of California) banded themselves together for the betterment of elementary school education.

"Our object", said their chairman, "is to attempt, through the labors of a series of carefully selected committees, to clearly define the purpose which should regulate the teaching of each of the several elementary school studies, and in accordance with that purpose reduce each of these studies to its lowest terms by eliminating all lessons and parts of lessons which do not specifically contribute to that purpose, and to study the best

1. Committee:
   Myrtle Collier, Chairman, Southern Branch. University of Calif.
   Katherine Spiers, Secretary, " " " " "
   Dr. A. W. Plummer, Los Angeles. Bertha R. Hunt, Santa Monica.
   Dr. W. H. Snyder, " " . Ruth Smart, Long Beach.
   Dr. A. H. Sutherland, Los " . Rufus Mead, Pasadena.
   Berthile Barclay, Santa Ana. Frances Brown, Riverside.
   Ann Burnam, Pomona. Jessie Wilkinson, San Bernardino.
2. Los Angeles, Santa Ana, Pomona, Santa Monica, Long Beach, Pasadena, Riverside, Redlands, San Bernardino.

ways and means of attaining that purpose in the teaching of each subject."[1]

The Committee on Arithmetic, which is still operative, met at intervals throughout the years 1918 and 1919 and much valuable work was done. The writer, during the years 1919, 1920 and 1921, has extended the study and has compiled this report which follows closely the line approved by the Superintendents of the Southern Cities and the Committee on Arithmetic.

In collecting these data an effort was made to reach representative groups. The data of Questionnaires I and II were obtained through the cooperation of the students of the Teacher Training Department of the Southern Branch of the University of California; superintendents, principals, and teachers of Southern, Central, and Northern California schools;[2] schools in Arizona, and in Alaska. The large city, the small town, the rural district, and the remote outpost of civilization are represented.

The data of Questionnaire III was collected through personal interviews with, and letters sent to, large business concerns in Los Angeles.[3]

1. Moore, E.C: Address, Los Angeles State Normal School, February 9, 1918.
2. Grateful acknowledgment is made to a group of superintendents, principals, and experienced teachers, members of a seminar group in the Summer Session of the University of California, 1919, for valuable help given during the years 1919,1920. Without their help this work could not have been carried on.
3. The writer thanks Dr.A.W.Plummer, Principal of the Twenty-ninth Street School, Los Angeles, for permission to use data collected by him and embodied in this report.

The problem throughout is a positive one. It is to determine what arithmetic men and women actually do use, what operations are employed, what figuring is actually done, and what insight and skills are required by business men and women.

Questionnaire I was sent to the general public: business and professional men and women, merchants, shop keepers, day laborers, ranchers, miners, cattle men, the leisure class, the home keeper. The time of collecting covers a period of one and one half years. The purpose of this questionnaire was to discover the size of the numbers used in the four fundamentals with whole numbers, fractions, and decimals, and the type of arithmetic used in daily life.

Questionnaire II[1] was sent to parents of pupils in the upper grammar grades and in high schools. The teachers of many schools, and the student-teachers of the Training School of the Southern Branch of the University of California collected these data. The questions were filled in at different ten-day intervals throughout a year. The purpose was to determine what figuring is actually done by people from day to day, and what arithmetical topics are in daily use.

The data of Questionnaire III are compiled from the re-

1. This questionnaire follows the line of those of Dr. Jessup and Dr. Coffman in 1913.

plies of such prominent Los Angeles business concerns as the following: Lewellyn Iron Works; Santa Fe Railway Company; Civil Service Commission; Los Angeles Creamery Company; Rivers Brothers, Wholesale Produce Company; H. Jevne Company, Grocers; A Hamburg and Sons, Merchants; R. L. Craig and Company, Importers and Wholesale Grocers; Hauser Packing Company; Los Angeles Planing Mill Company; Salt Lake Route; Goodrich Rubber Company; Cudahy Packing Company; Kahn-Beck Company; Bishop and Company; Sperry Flour Company; Los Angeles Ice and Cold Storage Company; Newmark Brothers; Howard Brokerage Company, Farm Products; Globe Grain and Milling Company; Los Angeles Public Library; Reynold E. Blight, Certified Public Accountant, others.

The purpose of this questionnaire was to determine business needs in arithmetic, to invite criticism of the school product, and to ask for recommendation toward improvement in the school course of study.

Questionnaire I.

The tables and charts on pages 70 to 84 inclusive, show, in tabular and graphic form, the results obtained from the questionnaire to the general public.

Questionnaire II.

The tables and chart on pages 87 to 90 inclusive, show, in tabular and graphic form, the results obtained from the questionnaire to parents.

Questionnaire III.

The totals and charts on pages 93 to 95 inclusive, show the judgment of a group of business men of Los Angeles.

CHAPTER II

QUESTIONNAIRE I.

(1500 printed or mimeographed copies sent out.
1136 replies.)

To the Public:

The purpose of this questionnaire is to find out how much arithmetic is used in every-day life. Do not state what you are able to use but what you actually do use. State the problem used in questions 1 to 10 inclusive. Do not sign your name.

1. Please state your occupation_____
   Answer question by underlining the numbers.
2. Do you personally have occasion to add columns of 2, 3, 4, 5, 6, or more numbers in height?
3. Do you personally have occasion to add columns of 2, 3, 4, 5, 6, or more figures in width?
4. Do you personally have occasion to multiply numbers of 2, 3, 4, 5, 6, or more figures?
5. Do you personally have occasion to multiply by numbers of 2, 3, 4, 5, 6, or more figures?
6. Do you personally have occasion to divide numbers of 2, 3, 4, 5, 6, or more figures?
7. Do you personally have occasion to divide by numbers of 2, 3, 4, 5, 6, or more figures?
8. How many of the following fractions do you personally have occasion to use: halves, thirds, fourths, fifths, sixths, sevenths, eighths, ninths, tenths, twelfths, sixteenths?
9. Do you personally have occasion to use decimals of 2, 3, 4, or more places?
10. Do you personally have occasion to compute simple interest?___
11. Do you yourself compute percentage,____
    (a) when you are paying taxes?_____
    (b) when you are paying commission?_____
    (c) when you are estimating profit and loss?_____
    (d) when you are shopping?_____
    (e) when you are paying insurance?_____

Note: This form was used by the Committee on Arithmetic, Southern California Cities.

	Persons Replying	Question 2						Question 3.					
		2	3	4	5	6	More	2	3	4	5	6	More
Accountants----	15	15	15	15	15	15	7	14	14	15	15	14	10
Attorneys-------	19	19	19	19	19	19	10	19	19	18	19	19	10
Bankers---------	37	37	37	37	37	36	16	37	37	36	34	33	24
Bookkeepers Stenographers---	75	75	70	69	68	62	44	75	72	69	55	44	29
Contractors-----	43	42	42	41	39	34	13	41	41	39	34	40	10
Day Laborers ---	54	53	51	49	42	36	17	52	47	42	30	21	7
Doctors, Dentists -------	25	24	24	24	22	19	12	24	24	23	15	8	3
Designers-------	9	9	9	9	6	6	1	9	9	9	7	6	0
Civil Engineers	23	23	23	23	15	13	11	23	22	22	19	16	8
Foremen---------	11	11	11	11	11	11	6	11	11	11	9	6	3
Farmers----------	108	105	104	101	94	84	50	102	101	89	67	48	28
Housewives------	237	194	190	173	157	133	74	220	213	177	86	30	19
Janitors--------	21	21	21	20	15	11	5	21	21	21	21	19	4
Librarians------	13	13	13	13	13	13	11	12	10	10	9	9	3
Machinists------	18	18	18	18	18	16	8	18	18	15	9	7	3
Managers---------	26	26	26	25	25	25	20	26	25	25	21	19	7
Merchants-------	156	152	155	144	134	134	80	148	148	134	119	83	35
Real Estate Men--	11	11	11	11	11	11	7	11	11	11	9	8	5
Teachers, Students -------	42	42	41	39	39	34	23	40	29	36	20	13	7
Miscellaneous	193	187	176	165	156	152	77	181	177	162	126	96	49
TOTAL	1136	1077	1056	1005	936	864	492	1034	1049	964	724	539	264

	Persons Replying	Question 4						Question 5					
		2	3	4	5	6	More	2	3	4	5	6	More
Accountant----	15	15	15	15	15	14	6	15	15	14	8	6	4
Attorneys -----	19	19	19	19	19	19	13	18	19	16	16	16	9
Bankers--------	37	33	30	28	21	16	6	25	18	16	13	10	4
Bookkeepers, Stenographers--	75	68	66	48	46	37	20	68	62	43	29	20	11
Contractors----	43	43	42	41	34	23	6	39	36	33	19	15	4
Day Laborers---	54	47	46	41	24	14	5	46	38	27	14	10	3
Doctors, Dentists-------	25	23	22	6	7	7	3	22	18	11	6	3	1
Designers------	9	9	9	9	5	3	0	9	8	6	3	2	0
Civil Engineers	23	23	22	20	17	14	6	23	22	20	18	14	6
Foremen--------	11	10	10	8	6	4	3	11	11	8	5	4	3
Farmers--------	108	104	104	97	66	48	19	95	83	65	38	26	12
Housewives-----	237	230	198	126	64	34	18	216	164	79	30	17	10
Janitors--------	21	21	21	19	15	8	3	21	17	7	7	5	1
Librarians-----	13	12	9	6	6	6	4	12	9	8	7	5	1
Machinists-----	18	18	18	18	12	9	2	18	16	11	6	6	2
Managers--------	26	24	24	21	14	10	3	24	20	15	13	9	2
Merchants------	156	148	149	125	85	73	27	142	123	85	60	46	22
Real Estate Men	11	11	11	11	7	6	0	11	8	5	3	3	0
Teachers, Students-------	42	40	38	33	26	17	8	36	31	19	8	10	6
Miscellaneous--	193	129	178	155	124	88	54	178	155	123	61	42	31
TOTAL----------	1136	1077	1031	846	613	450	206	1029	873	611	364	269	132

	Persons Replying	Question 6						Question 7					
		2	3	4	5	6	More	2	3	4	5	6	More
Accountants----	15	13	13	13	13	6	4	15	15	13	6	5	4
Attorneys------	19	19	19	19	19	17	18	19	19	18	14	14	9
Bankers--------	37	23	19	18	13	11	6	21	15	11	4	4	0
Bookkeepers, Stenographers--	75	69	59	51	42	48	8	55	41	30	27	27	10
Contractors----	43	40	39	31	26	16	7	40	35	28	23	20	9
Day Laborers---	54	45	43	40	24	16	4	45	37	21	9	7	3
Doctors, Dentists-------	25	22	20	14	9	9	5	21	17	9	3	3	1
Designers------	9	9	9	9	6	5	0	8	4	4	3	3	0
Civil Engineers	23	23	22	20	16	15	2	23	23	21	16	15	12
Foremen--------	11	11	11	11	8	5	3	11	11	8	5	5	3
Farmers--------	108	100	99	89	61	48	18	98	77	50	28	19	10
Housewives-----	237	203	181	149	60	31	17	144	106	28	11	8	8
Janitors-------	21	21	20	15	9	6	3	21	18	8	6	4	2
Librarians-----	15	12	9	6	3	3	1	12	9	5	3	1	0
Machinists-----	18	17	17	16	11	9	3	17	13	8	5	5	2
Managers--------	20	21	20	19	15	14	4	21	18	17	15	14	3
Merchants------	156	133	111	86	70	51	23	135	114	67	46	34	9
Real Estate Men	11	11	11	11	8	7	1	11	10	5	3	0	0
Teachers, Students-------	42	37	38	34	25	20	9	36	29	19	10	10	5
Miscellaneous--	193	164	157	146	114	68	40	169	143	110	75	50	27
TOTAL----------	1136	993	922	805	561	399	166	930	768	491	315	248	117

	Persons Replying	Question 8										
		1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9	1/10	1/12	1/16
Accountans----	15	14	8	14	11	8	3	10	3	13	7	6
Attorneys-----	19	19	19	19	19	16	15	15	15	16	17	13
Bankers-------	37	31	26	32	25	23	7	26	6	20	7	4
Bookkeepers, Stenographers-	75	63	56	61	38	29	18	37	17	34	23	21
Contractors----	43	43	36	41	24	19	11	23	15	27	24	13
Day Laborers---	54	46	24	39	18	16	12	25	11	22	12	9
Doctors, Dentists-------	25	23	13	20	10	9	6	10	6	10	5	6
Designers------	9	9	5	7	4	2	2	7	2	4	2	5
Civil Engineers	23	20	14	20	14	11	10	7	10	14	13	13
Foremen--------	11	11	10	9	6	6	4	8	4	7	2	2
Farmers--------	108	49	40	46	23	14	9	17	9	18	11	10
Housewives-----	237	183	128	141	65	50	33	46	23	39	17	12
Janitors-------	21	20	15	20	13	13	10	8	6	7	8	4
Librarians-----	13	12	5	10	8	3	2	2	2	7	1	0
Machinists-----	18	15	10	15	9	9	8	12	8	11	7	12
Managers-------	26	25	22	20	16	14	6	16	8	11	6	3
Merchants------	156	125	95	119	73	54	41	68	36	65	45	48
Real Estate----	11	11	8	9	6	5	0	3	3	0	6	5
Teachers, Students-------	42	41	35	39	22	18	11	23	8	20	9	6
Miscellaneous--	193	162	118	160	49	69	36	103	30	89	75	57
TOTAL----------	1136	921	687	841	453	388	244	466	222	434	297	249

	Persons Replying	Question 9				Question 10	
		2	3	4	More	Yes	No.
Accountants-----	15	15	15	9	5	14	1
Attorneys-------	19	17	18	8	5	19	0
Bankers---------	37	33	28	15	1	33	4
Bookkeepers, Stenographers---	75	65	56	30	15	52	23
Contractors-----	43	33	27	23	7	35	8
Day Laborers----	54	44	33	13	4	31	23
Doctors, Dentists--------	25	23	16	8	5	23	2
Designers-------	9	8	6	2	1	8	1
Civil Engineers-	23	22	22	21	8	11	12
Foremen---------	11	10	9	6	4	8	3
Farmers---------	108	84	53	42	24	65	43
Housewives------	23	149	31	14	4	160	77
Janitors--------	21	20	14	11	1	18	3
Librarians------	13	12	6	5	1	8	5
Machinists------	18	16	11	10	3	8	10
Managers--------	26	18	8	6	6	17	9
Merchants-------	156	130	87	69	25	99	51
Real Estate Men-	1	9	6	3	0	11	0
Teachers, Students--------	42	36	17	14	8	32	10
Misce llaneous--	193	165	137	70	52	66	127
TOTAL-----------	1136	909	599	379	179	724	412

	Persons Replying	Question 11									
		(a)		(b)		(c)		(d)		(e)	
		Yes	No	Yes	No	Yes	No	Yes	No	Yes	No
Accountants-----	15	8	7	14	1	14	1	10	5	13	2
Attorneys------	19	12	7	16	3	10	9	3	16	17	2
Bankers---------	37	20	17	25	12	22	15	15	22	20	17
Bookkeepers, Stenographers---	75	50	25	52	23	47	28	52	23	51	24
Contractors-----	43	25	18	26	17	30	12	28	15	30	13
Day Laborers----	54	20	34	21	33	17	37	16	38	34	20
Doctors, Dentists--------	25	10	15	8	17	10	15	11	14	7	18
Designers-------	9	2	7	7	4	5	5	4	3	6	5
Civil Engineers-	23	12	11	13	10	10	13	12	11	12	11
Foremen---------	11	5	6	6	5	5	6	8	3	4	7
Farmers---------	108	58	50	75	33	70	38	60	48	55	53
Housewives------	237	68	151	77	160	80	157	112	125	57	180
Janitors--------	21	11	10	12	9	13	8	14	7	10	11
Librarians------	13	3	10	3	10	4	9	8	5	4	9
Machinists------	18	9	9	8	10	9	9	8	9	9	9
Managers--------	26	14	12	18	8	16	10	14	12	18	8
Merchants-------	156	75	81	96	60	112	44	88	68	112	44
Real Estate Men-	11	8	3	11	0	10	1	6	5	6	5
Teachers, Students--------	42	19	23	21	21	23	19	25	17	28	14
Miscellaneous---	193	95	98	92	101	79	114	85	106	82	111
TOTAL-----------	1136	542	594	596	538	586	550	578	558	573	563

QUESTION I.

Please state your occupation.

Accountants	15
Attorneys	19
Bankers	37
Bookkeepers, Stenographers	75
Contractors	43
Day Laborers	54
Doctors, Dentists	25
Designers	9
Civil Engineers	23
Foremen	11
Farmers, Ranchers	108
Housewives	237
Janitors	21
Librarians	13
Machinists	18
Managers	26
Merchants	156
Real Estate Men	11
Teachers, Students	42
Miscellaneous[1]	193
Total	1136

1. Under miscellaneous are grouped those occupations which were reported 1, 2, 3, 4, or 5 times only, and which logically could not be included under any of the heads listed. Among these were: actors, (motion-picture) barbers, bakers, blacksmiths, brokers, cleaners, confectioners, dressmakers, dance hall and picture show managers, electricians, firemen, hotel and apartment house keepers, insurance agents, landlords, lawyers, miners, ministers, musicians, nurses, peddlers, photographers, policemen, railway employees, retired business men, sailors, soldiers, sheriffs, tailors, telephone operators, undertakers.

QUESTION II.

Totals and graphic representation of question number II :
Do you personally have occasion to add columns of 2, 3, 4, 5, 6, or more numbers in height?

Total number persons replying, 1136

Number using 2 addends------- 1077

" " 3 " ------- 1056

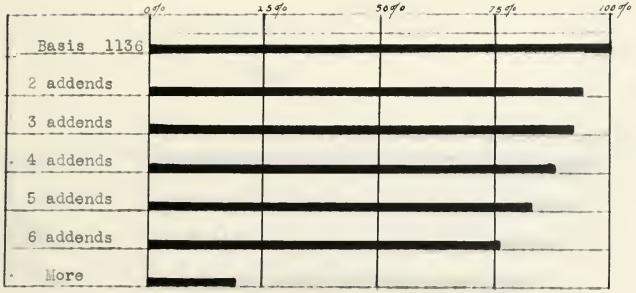
" " 4 " ------- 1005

" " 5 " ------- 936

" " 6 " ------- 864

" " more " ------- 492

GRAPH.

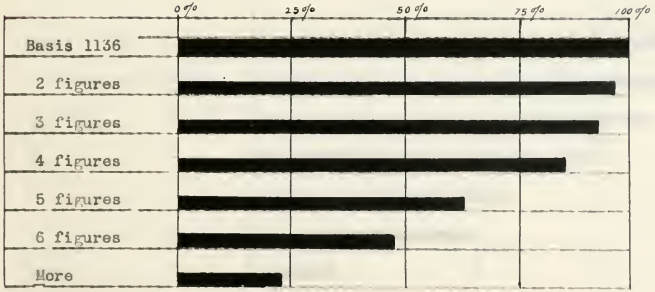


QUESTION III.

Totals and graphic representation of question number III.
Do you personally have occasion to add columns of 2, 3, 4, 5, 6 or more figures in width?

Total number persons replying, ----------	1136
Number using 2 figures in width ---------	1084
" " 3 " " " ----------	1049
" " 4 " " " ----------	964
" " 5 " " " ----------	724
" " 6 " " " ----------	539
" " more " " " ----------	264

GRAPH



QUESTION IIII.

Totals and graphic repreſent ation of question number IIII:
Do you personally have occasion to multiply numbers of 2, 3, 4, 5, 6,or more figures?

Total number persons replying -------1136

Multiplicand 2 figures ---------- 1077

" 3 " ----------1031

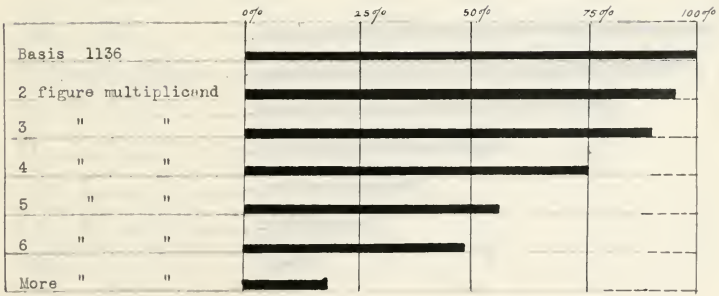
" 4 " ---------- 846

" 5 " ----------613

" 6 " ---------- 450

" more " ---------- 206

GRAPH.

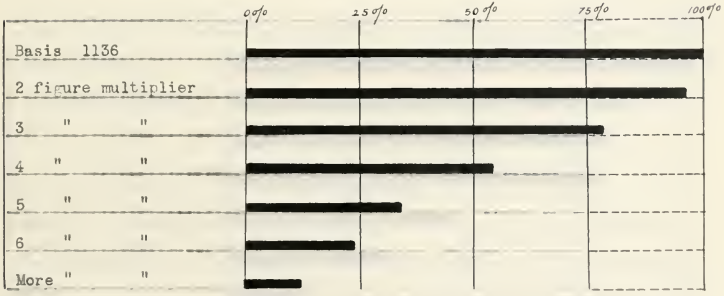


QUESTION V.

Totals and graphic representation of question number V:
Do you personally have occasion to multiply by numbers of 2, 3, 4, 5, 6, or more figures?

Total number persons reply ing	1136
Multiplier 2 figures	1029
" 3 "	875
" 4 "	611
" 5 "	364
" 6 "	269
" more "	132

GRAPH.

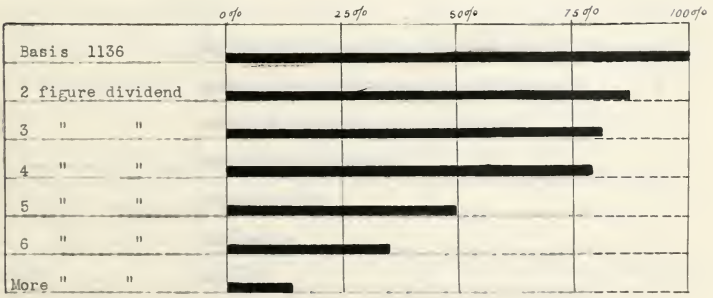


QUESTION VI.

Totals and graphic representation of question number VI:
Do you personally have occasion to divide numbers of 2, 3, 4, 5, 6, or more figures?

Total number persons replying			1136
Dividend	2	figures	993
"	3	"	922
"	4	"	805
"	5	"	561
"	6	"	399
"	more	"	186

GRAPH.



QUESTION VII.

Totals and graphic representation of question number VII:
Do y ou personally have occasion to divide by numbers of 2, 3, 4, 5, 6, or more figures?

Total number persons replying ----------1136

Divisor 2 figures ---------- 930

" 3 " ---------- 768

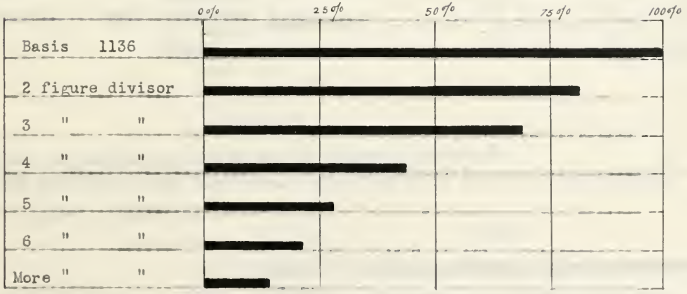
" 4 " ---------- 491

" 5 " ---------- 315

" 6 " ---------- 248

" more " ---------- 117

GRAPH.



QUESTION VIII.

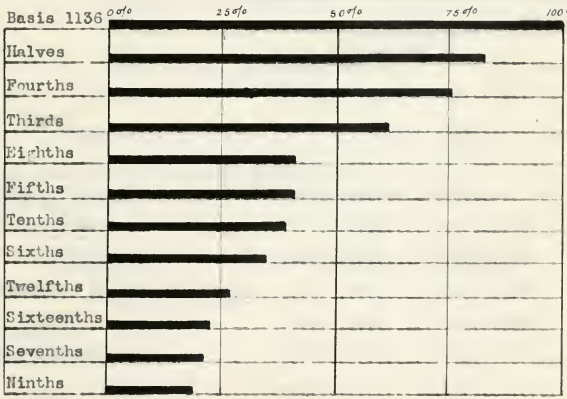
Totals and graphic representation of question number VIII.
How many of the following fractions do you personally have occasion to use: halves, thirds, fourths, fifths, sixths, sevenths, eighths, ninths, tenths, twelfths, sixteenths?

Total number persons replying -- 1136

TOTALS

Halves ------ 921
Fourths ----- 841
Thirds ------ 687
Eighths ----- 466
Fifths ------ 453
Tenths ------ 434
Sixths ------ 388
Twelfths ---- 297
Sixteenths -- 249
Sevenths ---- 244
Ninths ------ 222

GRAPH



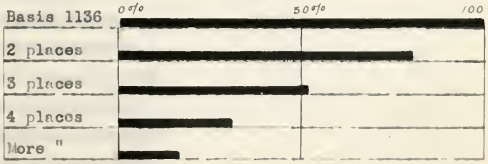
QUESTION IX.

Totals and graphic representation of question number IX.
Do you personally have occasion to use decimals of 2, 3, 4, or more places?

Total number persons replying - 1136.

Decimals 2 places --- 909
" 3 " --- 599
" 4 " --- 379
" more " --- 179

GRAPH



QUESTION X.

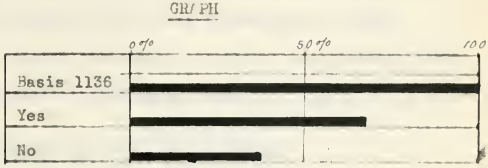
Totals and graphic representation of question number X.
Do you personally have occasion to compute simple interest?

Number persons replying, 1136

Yes ---------- 724

No ---------- 412

GRAPH

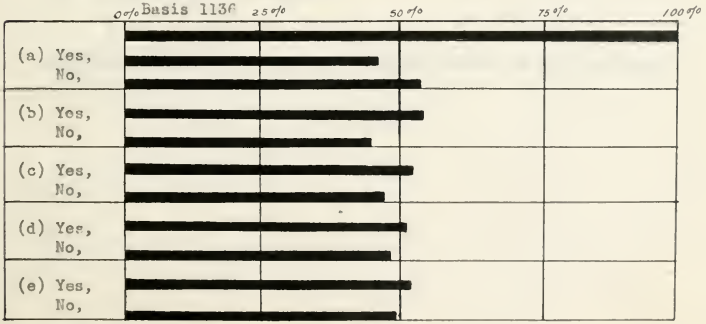


QUESTION XI.

Totals and graphic representation of question number XI.
Do you yourself compute percentage:

		Number persons replying - 1136	
(a)	When you are paying taxes?	Yes -- 542	No -- 594
(b)	" " " " commission?	Yes -- 598	No -- 538
(c)	" " " estimating profit and loss?	Yes -- 586	No -- 550
(d)	" " " shopping?	Yes - 578	No -- 558
(e)	" " " paying insurance?	Yes -- 573	No -- 563

GRAPH.



JUDGMENT.

After critically examining the tables and graphs of Questionnaire I, and giving due esteem to the problems reported -- all of which deal with buying, selling, measuring; paying wages, bills and debts; figuring averages, simple percentage and interest[1] -- the following deductions were made.

Minimum essentials with regard to size of numbers.

1. Addition - six addends, five figures wide.
2. Multiplication - multiplicand, five figures wide.
   multiplier, four figures wide.
3. Division - dividend, five figures wide.
   divisor, three figures wide.
4. Fractions - major group, halves, thirds, fourths,
   minor group, fifths, eighths, tenths.
5. Decimals - three places.

Essential computations.

1. Figuring simple interest.
2. Figuring percentage.

1. See Essentials in Problems, pages 108-109 of this study.

CHAPTER III.

QUESTIONNAIRE II.

(500 mimeographed copies sent out;
405 replies.)

To Parents:

The purpose of this investigation is to determine how much arithmetic is used in every-day life.

Will you please tell your child for each of ten consecutive days what you have done with numbers during the day? Kindly have the child place a cross (x) opposite the topic used, and note the problem.

Please state your occupation.

	1st Day	2nd Day	3rd Day	4th Day	5th Day	6th Day	7th Day	8th Day	9th Day	10th Day
Addition of Fractions										
Multiplication of Fractions										
Subtraction of Fractions										
Division of Fractions										
Cash Checks, or Bills										
Simple cash Accounts, or Family Expense Accounts										
Proportion										
Stocks - - Dividends										
Bonds										
Banking										
Paying in Part Payments										
Square Measure										
Volume										
Board Measure										
Drawing to Scale										
Graphs										
Land Measure										
Trade Discount										

QUESTIONNAIRE II.	Persons Replying	Banking Paying in Part Payments	Square Measure	Volume	Board Measure	Drawing to Scale	Graphs	Land Measure	Trade Discount
	Number of times used in 10 days.								
Blacksmiths-----	4	16	0	0	0	2	6	0	0	6
Bookkeepers-----	19	95	56	11	0	0	0	0	0	91
Carpenters------	19	26	18	103	11	154	59	0	5	17
Civil Engineers	5	11	3	3	1	3	7	18	4	2
Creamerymen-----	5	4	0	0	1	0	0	0	0	3
Electricians----	6	27	6	8	0	11	8	0	0	17
Housewives-------	125	102	56	19	41	15	18	0	9	30
Laundrymen ------	3	24	0	0	0	10	10	0	0	15
Mail Carriers----	3	10	1	1	0	0	1	0	0	0
Mechanics-------	26	25	47	84	34	51	78	0	4	37
Merchants-------	41	138	121	49	3	45	12	0	0	144
Miscellaneous---	63	150	56	71	61	79	40	7	50	82
Photographers---	4	6	5	0	0	0	4	0	0	26
Plumbers--------	3	13	0	0	0	0	0	0	0	0
Railroadmen-----	6	13	4	0	0	0	0	0	0	0
Ranchers--------	41	69	49	31	10	38	0	1	37	21
Real Estate Men-	9	55	28	0	0	0	2	0	0	3
Tailors---------	5	28	15	15	10	0	12	1	0	3
Teachers,Students	18	30	7	22	0	16	27	13	17	18
TOTAL----------	405	844	472	416	172	425	283	40	126	515

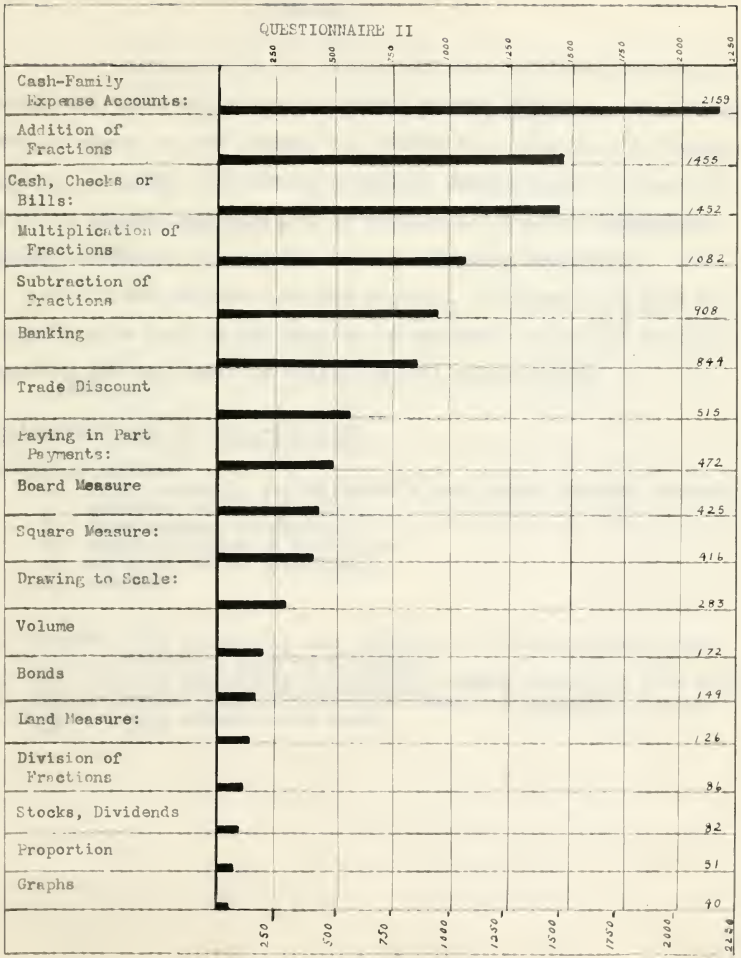
QUESTIONNAIRE II (continued)	Persons Replying	Addition of Fractions	Subtraction of Fractions	Multiplication of Fractions	Division of Fractions	Cash checks, or Bills	Cash and Family Expense Acct.	Proportion	Stock Dividends	Bonds
	Number of times used in 10 days.									
Blacksmiths------	4	19	10	15	0	21	37	0	0	3
Bookkeepers------	19	159	77	74	5	96	149	1	0	2
Carpenters-------	19	133	79	94	15	76	60	7	3	14
Civil Engineers--	5	24	20	34	4	9	16	0	0	1
Creamery Men-----	5	4	1	3	0	25	22	0	0	1
Electricians-----	6	35	11	11	3	17	27	0	0	1
Housewives-------	125	172	72	129	12	302	738	9	9	18
Laundrymen-------	3	16	15	18	1	15	16	0	0	0
Mail Carriers----	3	2	2	3	0	13	21	0	0	1
Mechanics--------	26	163	111	99	3	51	125	11	9	3
Merchants--------	41	281	229	234	9	255	288	9	16	10
Miscellaneous----	63	182	97	172	19	257	292	5	34	34
Photographers----	4	22	17	20	5	29	29	1	0	6
Plumbers---------	3	4	2	7	4	18	2	0	0	1
Railroad Men-----	6	24	25	25	4	15	16	0	0	3
Ranchers---------	41	53	42	44	2	145	146	4	7	15
Real Estate Men--	9	29	17	22	0	47	55	4	0	3
Tailors----------	5	20	9	7	0	21	35	0	2	15
Teachers, Students---------	18	133	72	71	0	40	95	0	2	16
TOTAL------------	405	1455	908	1082	86	1452	2169	51	82	149

QUESTION I

Please state your occupation.

Blacksmiths	4
Bookkeepers	19
Carpenters	19
Civil Engineers	5
Creamery men	5
Electricians	6
Housewives	125
Laundrymen	3
Mail Carriers	3
Mechanics	26
Miscellaneous[1]	63
Merchants	41
Photographers	4
Plumbers	3
Railroad men	6
Ranchers	41
Real Estate men	9
Tailors	5
Teachers, Students	18
Total	405

1. Under miscellaneous are grouped those occupations which were reported only once. (See note page 76, --Similar group--)


QUESTIONNAIRE II
250
500
750
1000
1250
1500
1750
2000
2250
1. Cash-Family Expense Accounts: 2159
2. Addition of Fractions 1455
3. Cash, Checks or Bills: 1452
4. Multiplication of Fractions 1082
5. Subtraction of Fractions 908
6. Banking 844
7. Trade Discount 515
8. Paying in Part Payments: 472
9. Board Measure 425
10. Square Measure: 416
11. Drawing to Scale: 283
12. Volume 172
13. Bonds 149
14. Land Measure: 126
15. Division of Fractions 86
16. Stocks, Dividends 82
17. Proportion 51
18. Graphs 40

JUDGMENT.

Reference to the tables and graph of Questionnaire II shows that the schools are not giving proper emphasis. Too little time is given to some topics, as, Simple Cash Accounts, or Family Expense Accounts; Cash Check, or Bills; Banking; and too much is given to others, as, Division of Fractions;[1] Volume; Proportion; Stocks, Dividends, and Bonds; Square and Board Measures.

The returns from the section, "Problems used from day to day", show that we are failing to utilize a wealth of practical material for problems[2] by following text books solely.

Arithmetic used in every-day life.

Essential topics.[3]

1. Cash Accounts, and Children's and Family Expense Accounts.
2. Addition of Fractions.
3. Cash Checks, or Bills.
4. Multiplication of Fractions.
5. Subtraction of Fractions.
6. Banking.

1. People often think they are dividing by a fraction when they take a fractional part of a number.
2. See section, Essentials in Problems, pages 108-109 of this study.
3. Let the reader bear in mind that these are essential topics, not the only topics to be used.

CHAPTER IV

QUESTIONNAIRE III.

(100 letters or personal interviews.
51 replies in whole or in part.)

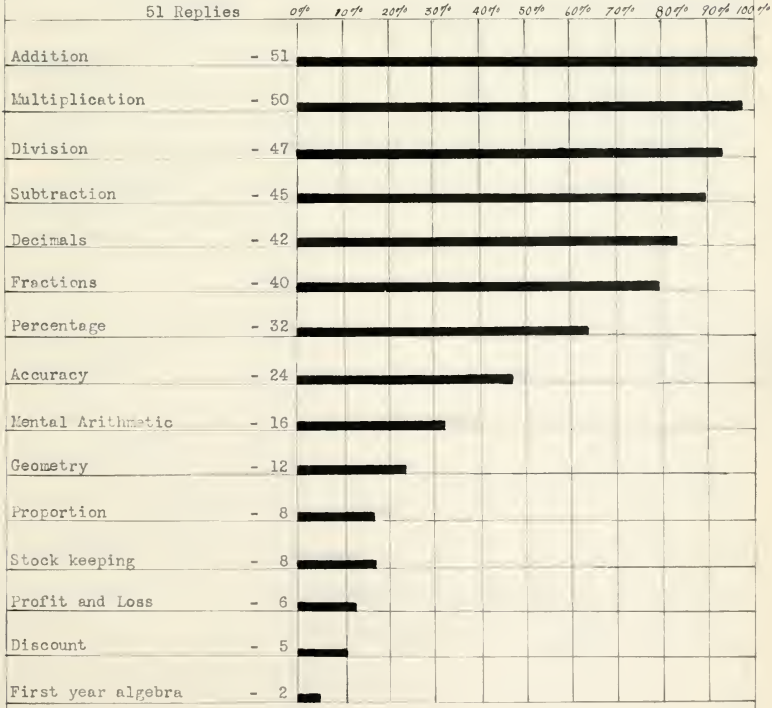
"To Business Men of Los Angeles:

Will you please assist our public school teachers, and thereby help the boys and girls, by giving the following questions careful consideration and sending your reports to the Committee on Arithmetic?

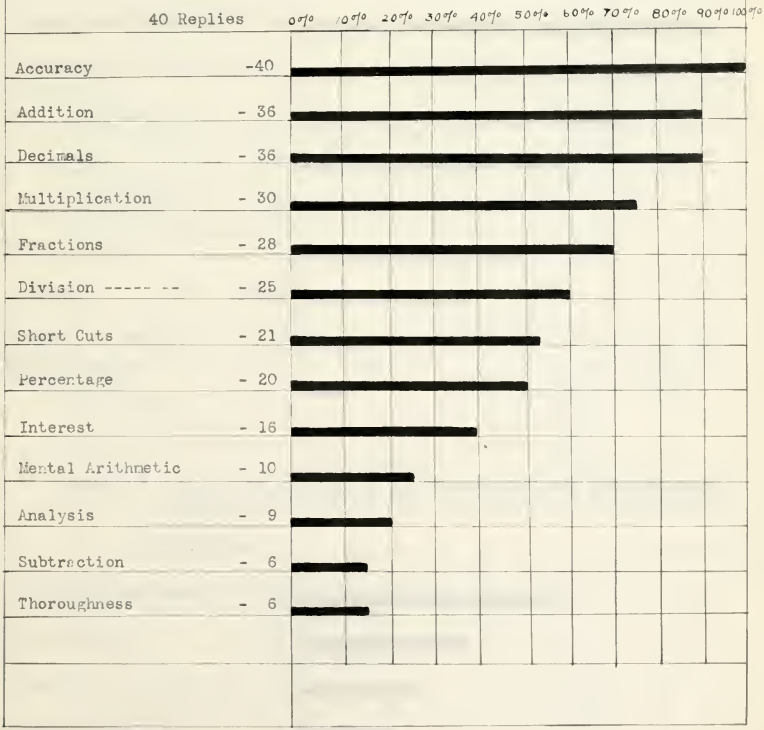
1. How much arithmetic should young people know when they enter your employment?
2. In what arithmetic work do you find them weak or unsatisfactory?
3. What suggestions do you make that may assist in correcting mistakes?
4. So far as it comes to your attention, what work in arithmetic is being taught that is of little or no value in your business?"

1. Note 3, page 65.

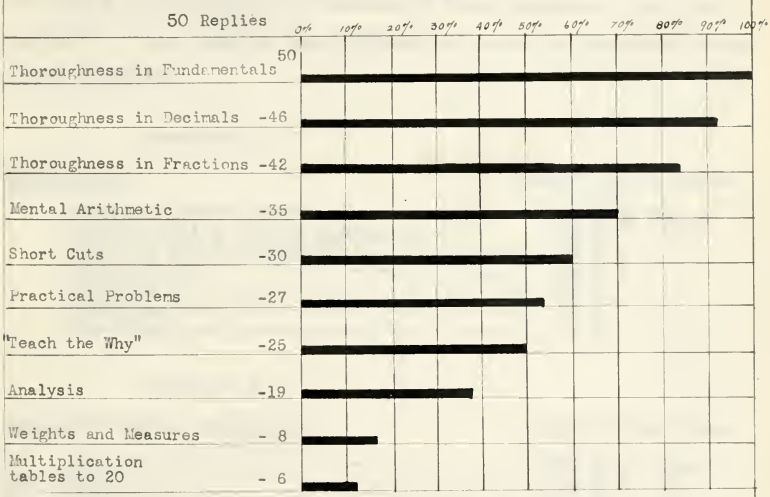
Question 1. How much arithmetic should young people know when they enter your employment?



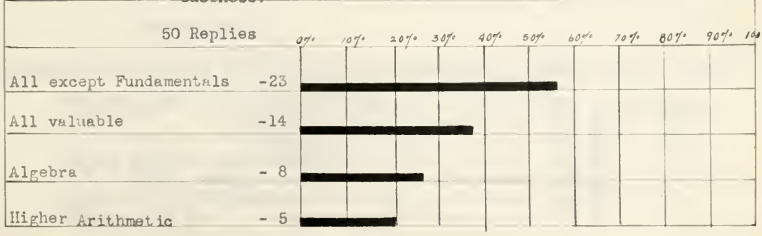
Question 2. In what arithmetic work do you find them weak or unsatisfactory?



Question 3. What suggestions do you make that may assist in correcting mistakes?



Question 4. So far as it comes to your attention, what work in arithmetic is being taught that is of little or no value in your business?



JUDGMENT.

Business firms demand skill. They ask for accuracy and speed in handling the fundament/: als whole numbers, fractions, decimals. They require thoroughness and power to interpret practical problems.

Question 1.

Essentials: Report of 51 Firms.

Addition (whole numbers)	100 % of Firms
Multiplication (whole numbers)	98 % " "
Division (whole numbers)	92 % " "
Subtraction " "	88 % " "
Decimals	82 % " "
Fractions	78 % " "
Percentage	62 % " "

Question 2.

Employees weak, unskilled: Report of 40 Firms

Accuracy	100 % of Firms
Addition	90 % " "
Decimals	90 % " "
Multiplication	75 % " "
Fractions	70 % " "
Division	62 % " "
Short Cuts	52 % " "

Question 3.

Suggestions toward improvement of teaching: Report of 50 Firms.

Thoroughness in fundamentals	100 % of Firms
" " decimals	92 % " "
" " fractions	84 % " "
Teach mental arithmetic	70 % " "
" short cuts	60 % " "
Practical problems	54 % " "
"Teach the Why"	50 % " "

Question 4.

Valueless arithmetic: Report of 50 Firms

All except fundamentals	46 % of Firms
All valuable	28 % " "
Algebra	16 % " "
Higher arithmetic	10 % " "

SUMMARY

ELIMINATION OF TOPICS.

Through a careful comparison of the data obtained by means of the three questionnaires, with the results, records and judgments collected under Chapter III, "Eliminations Which Have Been Suggested or Made," the following sweeping reduction in subject matter is recommended:

Eliminations to be made in Elementary Texts and Elementary Courses of Study.[1]

1. Apothecaries Weight.
2. Troy Weight.
3. Longitude and Time.
4. Furlong in Linear Measure.
5. Hand.
6. Dram in Avoirdupois Weight.
7. Surveyors' Table.
8. Fathom.
9. All problems in reduction, ascending and descending, involving more than two steps.
10. G. C. D. as a separate topic.
11. All initial common fractions except halves, thirds, fourths, fifths, sixths, eighths, ninths, tenths, twelfths, sixteenths, hundredths, thousandths. (Initial fraction is the fraction given for the solution of a problem.)
12. All work with L. C. M. except of very low denominations. (As a separate topic.)
13. Complex fractions.
14. Compound proportion.
15. Cases in percentage.
16. True discount.
17. Compound interest, except in simple savings accounts as re-invested money.
18. Problems in partial payments.

1. All of these topics are found in Texts or Courses of Study used in public schools during the past ten years.

19. Commission and Brokerage (as applied to stocks and bonds.)
20. Profit and Loss as a special topic.
21. Knot.
22. Partnership (as a special topic).
23. Cube Root.
24. All algebra, except such simple use of the equation as is directly helpful in arithmetic and in other subjects met with in the school life of the pupil.
25. Brackets, Braces, Vincula.
26. Cancellation, as a special topic.
27. Finding the whole when a fractional part is given.
28. Paper tables.
29. Gross and Great Gross.
30. Square (100 sq. ft. used in roofing).
31. Carpeting, Lumber Measuring, Papering, Plastering, Painting, as separate subjects.
32. Surveyors' Land Measure.
33. Foreign Money.
34. Indirect problems in simple interest. (Use the equation.)
35. Bank Discount.
36. Pyramids, Cones, Spheres.
37. Metric System. (To be learned as occasion for use arises.)
38. Initial decimal fractions of more than three places.
39. All problems whose content is outside the experience of the child.
40. Examples of this type: $5 + 3 \times 7 \div 6 - 4$.
41. All improbable problems.
42. All problems in which the part and its fractional equivalent are given to find the whole.
43. All so-called problems without number. /

ESSENTIALS IN SUBJECT MATTER.

If the results indicated in this study are to be attained, and if the returns from the three questionnaires, which definitely emphasize the judgments of experimental research workers along a like line, be considered valid basis for establishing the business and social usage of arithmetic, the following is the legitimate field of elementary arithmetic.

/ Work of Committee Appointed by ... Experiments Notes 1, 2 p. 49

Requirements:

A high degree of accuracy in solving problems involving:

1. Addition, six addends, five digits wide.
2. Subtraction, six digits wide.
3. Multiplication, multiplicand of five digits, multiplier of four digits.
4. Division, dividend of five digits, divisor of four digits.
5. Addition, subtraction, and multiplication of these fractions: halves, thirds, fourths, fifths, eighths, tenths.
6. Addition, subtraction, multiplication, and division of decimals to three places.
7. Simple interest.
8. Percentage. (avoid the indirect problem.)
9. Cash accounts, and children's and family expense accounts.
10. Banking.
11. Common measures.

Schools have made arithmetic unnecessarily difficult, when, in truth, the work should be simple and easy, as an ability to do practical problems is all that is required in life. The average individual needs to know how to add, subtract, multiply, and divide whole numbers and decimals; to add and subtract simple fractions; to find a fractional part of anything; to multiply a whole number by a fraction, perhaps a fraction by a fraction; to find percentages; and to be familiar with common measures. Our pupils should be able to master these necessary things, and master them thoroughly, by the end of the sixth year. Studies in elimination and retardation which have been carried on in recent years show that a very great number of children do not complete the eight-year elementary school course. In view of this fact, it is

of vital importance that they be taught the arithmetical operations required in life as early as possible in their school career. The two years' time thus saved might with profit be utilized in giving boys and girls more extensive acquaintance with problems connected with social, industrial and civic life. Where desired, a unified course made up of arithmetic, algebra, and geometry, all worked out to fit life-needs, might follow the six-year arithmetic course.

In this brief and partial survey of the outstanding needs of the public in arithmetic and the relation of these needs to school instruction, many things have been omitted which deserve consideration, and many questions have been suggested to which no definite answers have been given. Much of the material here presented is already familiar to the forward-loking teacher who has made a study of arithmetic. The aim has not been to exhaust the subject, but to point out the real issues and to group the facts established around a center of progress. It takes many people working together to map out a program for the teaching of any subject. The elaboration of the following course of study or the pruning of it as the critics judge fit must be left to those interested teachers who will use their classrooms as trying-grounds, who are ready to study with new interest and thoroughness every aspect of the subject, and who are willing to weigh all values on the scales of public need.

5. Playing dominoes. (a) Matching.
   (b) Counting by 5s.
6. Time. (a) Making a clock face.
   (b) Hours, 9, 12, 2, etc.
7. Games. scoring, (a) Bean bag.
   (b) Ring toss.
   (c) Nine pins.
   (d) Hook it.
   (e) Guessing games.
   (f) Building up numbers, using all possible combinations.
8. Number stories.

Through such activities, the minimum number work of the pupil should be:

First Grade -

1. (a) Count to 20 concretely.
   (b) Count to 20 abstractly. (Symbols are to be used after the numbering knowledge has been obtained by the use of objects in work and play.
2. Group objects by 2's and 5's to 20. Count by 2's and 5's to 20. (Grouping and counting symbolized with written words and with digits.
3. Divide groups of objects into 2's, 3's, and 4's to 12.
4. Use term halves when groups of objects are divided into two equal parts. (Not more than 12 objects to be used.)
5. Emphasize the relationship between quantities by means of objects. (Suggestion: Relationship between inch and foot, pint and quart.)
6. Denominate numbers.
   Measurements -
   12 inch = 1 ft.
   2 pint = 1 qt.
   12 things = 1 doz.
   5¢ = 1 nickel.
   10¢ = 1 dime
   2 nickels = 1 dime.
7. The addition of halves and halves.

Second Grade -

1. Review the work of first grade.
2. Continue counting concretely by 2's and 3's to 24; by 4's to 40; by 5's and 10's to 50.
3. Present the thirty-three combinations, concretely, whose sums are 12 or less. Aim toward an abstract and automatic mastering of these.
4. Division of groups of objects into 2's, 3's, 4's, 6's, to 24.
5. Divide groups of objects into 2, 3, 4, 6, equal parts. Maximum 12 objects.
6. Use terms halves, fourths when objects are divided into two or four equal parts.
7. Column addition, two digits wide. (Sum of each column less than 10.)
8. Column subtraction, two digits wide. (Each digit in the minuend to be greater than the corresponding digit in the subtrahend.)
9. The addition of fourths to fourths. (In hand work problems.)
10. Denominate numbers.
    25¢ = 1 quarter.
    60 min. = 1 hour.
    7 days = 1 week.

Aim in the Third and Fourth Grades -

Since it is in the third and fourth grades that habits of accuracy or inaccuracy are formed in the basic work in mathematics, the aim in these grades should be the automatic mastery of the forty-five combinations in addition, and the corresponding number facts in subtraction; the multiplication tables through 10 x 10, and the corresponding number facts in division. In addition to this the child should learn column addition and subtraction involving the adding-in and the taking-from processes; multiplication with a multiplier of two or three digits, and division with a divisor of two digits.

Third Grade -

1. The forty-five combinations in addition and subtraction made automatic.
2. Column addition three digits wide, four addends. (With the adding-in process.)
3. Subtraction with three digits wide. (With the taking-from process.)
4. Multiplication tables - 2's, 3's, 4's, 5's, 6's, 10's, through 6 times the number multiplied.
5. Division, corresponding to the combinations in the multiplication tables, and also with remainders.
6. Denominate numbers.
   3 ft. = 1 yd.
   4 qt. = 1 gal.
   50¢ = one-half dollar.

Fourth Grade -

1. Continue work of third grade.
2. Addition, four digits wide and four addends, also three digits wide and five addends.
3. Subtraction, four digits wide.
4. Complete the multiplication tables through the 10's. (10 x 10) Multiplicand 4 digits wide, multiplier 2 digits wide. (Using dollars and c cents.)
5. Division: Dividend not more than five digits; divisor not more than two digits.
6. Denominate numbers.
   16 oz. = 1 lb.
   10 dimes = $1.
   100 cents = $1.

Fifth Grade -

The aim in the fifth grade should be the mastery of common fractions.

1. Continue drill on the four fundamentals with whole numbers.
2. Fractions.
   (a) Continue fraction work of the previous grades.
   (b) The addition of fractions in the following order of groups:
      Halves and halves.

Halves and fourths.
Halves, fourths, eighths.
Halves, thirds.
Halves, thirds, sixths.
Halves, thirds, sixths and twelfths.
Halves, thirds, fourths, sixths and twelfths.
Fifths and tenths.

(c) Reduction of fractions when necessary in addition and subtraction.
(d) Addition and subtraction of mixed numbers. In addition, two digits wide, three addends.
(e) Multiplication of fractions - 1/2, 1/3, 2/3, 1/4, 3/4 each repeated a given number of times. (Example, 3 x 1/2 = 3/2.) 1/2, 1/3, 2/3, 1/4, 3/4, 1/10 of a group. (Example 2/3 of 18.) Multiplication of simple mixed numbers (In playing store, etc.)
(f) Division of fractions. How many 1/2's, 1/3's, 1/4's, 3/4's, 1/10's in whole numbers. Example, how many 3/4's in 6, or 6 ÷ 3/4 = ? Develop through the concrete, use abstractly.

3. Denominate numbers.
   (a) Review - Continue the work of previous grades.
   (b) New - 24 hr. = 1 day.

Sixth Grade -

1. Drill on the four fundamentals in whole numbers and fractions.
2. Decimal fractions.
   (a) The decimal idea.
   (b) The four fundamentals in decimals, with a limit to two decimal places in initial decimals (Stress dollars and cents).
3. Per cent.
   1/100 = .01 = 1%. Nothing new.
4. Find the per cent of a number.
5. Find what percent one number is of another (18 is what per cent of 20?)
6. Denominate numbers.
   (a) Review denominate numbers through problem work.
   (b) New work to be taken up through problems.

100 lbs. = 1 cwt.
2000 lbs. = 1 ton.
144 sq.in. = 1sq. ft.
9 sq.ft. = 1 sq. yd.
160 sq.rd. = 1 Acre.
60 sec. = 1 minute.
365 days = 1 year.
12 mo. = 1 year.

Seventh, eighth and ninth grade mathematics should be a unified course made up of arithmetic, algebra, and geometry, and presented in such a way that there will be no definite break from arithmetic into algebra.

Seventh grade -

1. Drill for speed and accuracy with whole numbers, fractions and decimals.
2. The application of numbering to real life needs:
   (a) Social.
   (b) Industrial.
   (c) Civic.
3. Discount - as per cent.
4. Interest.
   (a) Money.
   (b) Investments.
5. Commission - in connection with vocational guidance.
6. Taxes - local; in simple form in connection with civics.
7. Personal accounts.
8. Banking:
   (a) How to make out a deposit slip.
   (b) How to write a check.
   (c) How and why to fill the stub.
   (d) When a check should be cashed.
   (e) How to stop payment on a check.
   (f) How to indorse a check.
   (g) How to indorse a note.
   (h) How to write a negotiable note.
   (i) How to compute interest.
   (j) Importance and purpose of savings banks.
   (k) Importance and purpose of commercial deposit banks.
   (l) How to open an account.
   (m) How to secure a bank draft.
   (n) How to use a bank draft.

9. The solution of the simple equation in algebra.
   (a) Definition of equation.
   (b) Making equations.
   (c) Solving equations by:
       Adding the same amount to both sides.
       Subtracting the same amount from both sides.
       Multiplying both sides by the same amount.
       Dividing both sides by the same amount.
   (d) Use of the parenthesis in equations.
   (e) Positive and negative numbers in the equation.
10. Intuitive geometry based upon shape, size and location of objects.
   (a) The rectangle.
       (x) Perimeter.
       (y) Area.
   (b) Triangle.
       (x) Angles.
       (y) Similar triangles - in solution of problems only.
       (z) Construction of similar triangles.
11. Use of the compasses in drawing straight lines, dropping a perpendicular from a point to a line, erecting a perpendicular at a given point in a line, constructing equal angles.
12. Use of the protractor in measuring angles.

Eighth Grade -

1. Further development of Seventh grade work.
2. Graphs.
3. The right triangle - Theorem of Pythagoras.
4. Relation of opposite angles and of the angles made by a transversal cutting parallel lines.
5. Cubical contents of rectangular prisms.
6. Cubical contents of cylinders.

The work in numbering must be a unit from the first grade through the eighth (Example: After fractions have been learned there is nothing new in decimals but the form of writing. Per cent is only a particular common fraction. 1/100, or a particular decimal, .01 = 1/100 = 1%.)

Much time will be saved by this unity of work. Further-

more, the child will be able to master the subject of arithmetic if this simplicity of the subject is shown.

ESSENTIALS IN PROBLEMS.

1. All problem work should be such as will develop the child's ability in numbering -- that is, the content of the problem must be within the comprehension of the child.

2. Children should make and solve problems growing out of their own experiences or environments, covering the number work as outlined for the various grades, using the suggested topics of activity as a guide.

3. In problem solving the five steps are to be learned:
   a. State the problem clearly, or read it understandingly.
   b. Pick out the unknown fact or facts.
   c. Choose the form of relating the known facts (add, subtract, multiply, or divide) in order to determine the unknown fact or facts.
   d. Solve. (This involves the mechanics of the subject.) The child should approximate his result before solving. The result should always be checked by the pupil.[1]

The returns from that section of Questionnaires I and II, "State the problem," indicate that the stereotyped problems in the majority of textbooks in use to-day are not problems met in life. The following list is compiled from these returns, and from personal observation of the things children really do.

Problem Material: Real grocery bills, making change, games, value of a cafeteria meal, measurements; inch, foot, yard, mile, (fraction of), acre, dozen, (fraction of) liquid (pints,

1. From "Minimum Essentials," 1921. (to be published.)

quarts, gallons,) time, U. S. Money, pound, ton, averages, home garden problems, errands, cost of heating and lighting a home, reading meters, cyclometer, pedometer; wages, labor questions, chicken business, plotting gardens and courts for games, measuring land and city lots, cost of furnishing a home, cost of building a garage -- cement, lumber, labor, paint, draw to scale, insure,-- borrowing and lending money, commissions, insurance, local taxes, shopping, dairy business, interest, bond coupons, banking (every phase met with by depositors), income tax, discount in shopping, household accounts, budgets, sugar and cotton industries, freighting, fire department, city market, real estate, excavations, road and street building and repairing, transportation -- street car, railroad, ship,-- percentages in games, races, field meets,---- others provided they meet this standard:

"The good textbook and the good teacher scrutinize every task they assign to make sure that it fits the pupil for life. They seek to find, for every arithmetical principle and fact, the real affairs to which it applies and with which it should be connected in the pupil's mind."[1]

1. Thorndike, Edward Lee: New Methods in Arithmetic. 1921.

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APPENDIX.

CRITICISM AND SUGGESTIVE CHANGES FOR THORNDIKE'S ARITHMETICS.[1]

Book One. Part One. Third grade.

Page 20. No's. 12, 13, 14 have little or no meaning to 8 year old children. Same is true of No. 13 page 22.

Eliminate all "counts" except from 0. See pages35, 37, 79, 80, 92, 118, 100 (first 10), 120.

Pages 42, 43, and 135. "------ which number means dollars etc.". There is only one number representing dollars and cents. The word number should be replaced by part or figures.

Page 44.-- "So increase the 7 to 17." Impossible. "Increase" should be eliminated and what is actually done (added) stated; as, add 10 to 7 etc.

Page 46. Eliminate No's. 7, 8, 9, and 10 as they are of no value to third grade pupils.

Pages 65, 106, 107, 108 and 111 are not for third grade.

Pages 66, 70, and 71. "Write 3 in the tens column." There is no tens column. Better,-- "in the tens' place."

Pages 74 and 75 and No's. 14 to 18 inclusive, might do for fifth or sixth grade, but not for third.

Page 87. "Write 4 over the 0 of 30." Write 5 over the 0 of 90.

1. Note page 65, note 3.

Pages 88, 89, 90 are better for the fourth grade.

Book One. Part Two. Fourth grade.

For lower grades, especially, the form of the fraction should have a horizontal line; as, $\frac{1}{3}$ instead of 1/3. $6\frac{1}{3}$ vs. 6 1/3.

Page 127. No. 6, is psychologically incorrect. Better divide by 2, 3, etc. as we would if using 875.

Page 133. What is the object? Eliminate.

Page 142, 143, 144, 145, 173 better be placed in Book Two - Part Two.

Page 150. Why such problems for 10 year old pupils? Eliminate.

Page 159. No child below the high school can answer No's. 5 and 6, or do 14 and 15, or cut a pie into fifths. Eliminate

Page 166 No's. 5 and 8. "Write the 1 after the quotient ---." 1 over the divisor is a part of the quotient. "Place 1 over the divisor as a part of the quotient" is better.

Page 177. Change directions for dividing by a three figure divisor. Children cannot "think" three or more figures into a dividend.

Page 187. No. 2 better multiply by $\frac{3}{8}$ first and make but one addition. Page 191, same criticism.

Why page 199 with Fourth grade, or any other? Eliminate.

All the work of adding and subtracting mixed numbers and changing improper fractions and mixed numbers should be put in

fifth grade.

Book Two. Part One. Fifth grade.

Page 10. No's. 1, 2, 5, and 6, change "increase" to add 1, or $\frac{8}{8}$ etc.

Page 24, "Numbers like $\frac{1}{8}$ --", change to "fractions like $\frac{1}{8}$ etc. and fractions like $\frac{4}{8}$" etc.

Page 32. Eliminate counts, No's. 1, 2, and 3.

Page 72. "----- which numbers mean miles and." There is only one number in the product. Should be which part or figures mean miles, etc.

Page 74. Eliminate No's. 17, 20, 23, 29, and 33, also Page 82 No's. 1, 2, 3, 4, and 5.

Page 75. Why not state where to put the decimal point instead of requiring the teacher to tell where it belongs? The work with decimals, pages 67 to 90 and 99 to 113 inclusive should be in part two, for sixth grade, also the work with compound denominate numbers, pages 91 to 97 inclusive.

The fifth grade needs more work with fractions. If it be taken from the fourth grade work and put into Book Two, Part One, the work for both grades will be improved.

In multiplying a mixed number, or by a mixed number, the fraction should be used first.

Book Two. Part Two. Sixth grade.

Page 137. Eliminate complex fractions, except the aliquot parts of 100, or 1000.

Page 142. Eliminate No's. 3 and 6. The answer to No. 6 is nothing.

Pages 200 (Commission), 201 (Gains and Losses), 202 (Fixing Prices). 205 (Sharing in percents), 206 (Interest) and 207, should be put into Book Three, for 7th grade. The same is true of Mensuration, pages 221 to 231.

Pages 243, 111 (Commission). 244, 245, 246, and 248 shoul be placed in Book Three for 7th grade.

Taking from the fifth grade and giving to the sixth, and taking from the sixth and giving to the seventh, as indicated, will make both divisions of work within bounds of what can be done.

Book Three. Part One. Seventh grade.

Page 2. "-- adding thousandths --" should be number one.

" 2. "-- adding hundredths --" should be number two.

" 2. "-- adding ones ---" number three.

" 2. "-- adding tens ---" number four.

Page 6. Eliminate counts.

Page 13. Form of checking (no. 17) never in business.

Page 15. To divide by or multiply by a "number". What about fractions?

Page 16. "Multiply the integer and numerator." Substitute by for and.

There appears but little new work for the seventh grade. The percentage work, and the mensuration given for the sixth grade is needed for the seventh.

Book Three. Part Two. Eighth grade.

Work for the eighth grade is good but the quantity is out of proportion to that given for the seventh grade.